Functional Differential Equations and Applications

Research Workshop of The Israel Science Foundation
29.8-2.9.2010
Ariel, Israel
Estimates of Zones where All Solutions of Functional Equations Change their Signs

Alexander Domoshnitsky and Roman Yavich
Ariel University Center of Samaria, Ariel, Israel

We obtain assertions about possible sizes of zones of positivity of solutions to functional equations. Our approach is based on theorems about estimates of the spectral radius. Simple coefficient tests are proposed.

Maximum Principles for One of the Components of Solution Vector for System of Functional Differential Equations

Alexander Domoshnitsky
Ariel University Center, Ariel, Israel, adom@ariel.ac.il

The method to compare only one component of the solution vector of linear functional differential systems, which does not require heavy sign restrictions on their coefficients, is proposed in this talk. Necessary and sufficient conditions of the positivity of elements in a corresponding row of Green's matrix are obtained in the form of theorems about differential inequalities. The main idea of our approach is to construct a first order functional differential equation for the nth component of the solution vector and then to use assertions about positivity of its Green's functions. This demonstrates the importance to study scalar equations written in a general operator form, where only properties of the operators and not their forms are assumed. It should be also noted that the sufficient conditions, obtained in this talk, cannot be improved in a corresponding sense and in many cases does not require any smallness of the interval [0; ω]; where the system is considered.
Observability criteria and design of observers for a class of linear discrete-continuous systems

Grigory Agranovich
Ariel University Center of Samaria, Department of Electrical and Electronic Engineering, Israel

Abstract

The studied linear discrete-continuous system contains two coupled subsystems: one with continuous-time dynamics, the other with discrete-time dynamics. Continuous-time dynamics are described by ordinary linear differential equations, whereas discrete-time dynamics are described by difference equations for the system state jumps at prescribed time instants. Systems of three following degrees of generality are considered: time-varying, time-periodical and time-constant discrete-continuous systems. Some significant observability properties of such systems are presented.

Kalman – like observability criteria are proposed for each of these classes of systems. Hautus-like eigenvalues observability criteria are developed for time-constant models. Discrete-continuous observer design method is developed. The method based on proposed solution of an eigenvalue assignment problem for a discrete-continuous constant parameters system.
Abstract

In finite populations, birth death processes can converge to an absorbing state, where some of the populations are extinct. This extinction point can be non-stable in the deterministic continuous description of the birth-death process.

The dynamics of birth death processes with extinction points that are unstable in the deterministic average description has been extensively studied, mainly in the context of the stochastic transition from the mean field attracting fixed point to the absorbing state. We here study the opposite case of a small perturbation from the zero population state and show that such perturbations can grow beyond the mean field attracting fixed point and then collapse back to the absorbing state. Such dynamics can represent, for example, the fast growth of a pathogen and then its destruction by the immune system. We study these dynamics using a spatially extended stochastic discrete predator – prey model.

We compute the prey extinction probability following the introduction of a limited amount of prey to the system, and show that when this probability is high, the loss of synchronization between the prey density in different regions in space leads to two possible dynamic regimes. The first regime is a directed percolation regime based on the balance between regions escaping the absorbing state and regions absorbed into it. The second regime is wave trains representing the transition of the entire space to the mean field stable positive fixed point. These two regimes differ from the mean-field results and do not rely on a low number of either predators or preys. This work creates a direct bridge between the predator prey dynamics, directed percolation and wave trains.
Asymptotic solutions to Vlasov-Maxwell equations for relativistic short beam

Frank Assous
Ariel, University Center, Ariel, Israel

Charged particle beams and plasma physics problems are extensively used in Science and Technology. If we consider collisionless plasma or non-collisional beams, one of the most complete mathematical models is the time-dependent Vlasov-Maxwell system of equations. However, the numerical solution of such a model, which is unavoidable in many situations, requires a large computational effort. Therefore, whenever possible, we take into account the particularities of the problem to derive asymptotic approximate models leading to cheaper simulations.

In this talk, we will derived an asymptotic paraxial model as an approximation of the time-dependent Vlasov-Maxwell equations for high energy short beams. The model is derived by introducing a frame which moves along the optical axis at the speed of light, so that the bunch of particles is evolving slowly in this frame. Then one considers a scaling of the equations which reflects the characteristics of the high energy short beam. Finally, we introduce a small parameter and we use asymptotic expansion techniques to obtain new paraxial models with a controlled accuracy.

The simplicity of the so obtained formulations allows to use a finite-difference discretization for the Maxwell equations. However, an open question remains: despite a theoretical convergence result, it is not always easy to determine which terms to retain in the asymptotic expansion to get a sufficiently precise but not too expensive model. Indeed, with this asymptotic approach, one can derive several approximate models, accurate up to first, second or third order. In a companion talk by I. Chaskalovic (this session), it will be proposed to use data mining techniques to try to answer to this question.
On Estimates of Alternating-Sign Solutions to Nonlinear Differential Equations.

Irina Astashova

Moscow State University of Economics, Statistics and Informatics, Moscow Lomonosov State University
ast@diffiety.ac.ru

**Keywords:** nonlinear ordinary differential equation of higher order, uniform estimates of solutions.

Uniform estimates and qualitative behavior of solutions to quasi-linear ordinary differential equations of the higher order are described. In particular, to the equation

\[ y^{(n)} + \sum_{j=0}^{n-1} a_j(x) y^{(j)} + p(x) |y|^k \text{sgn}y = 0 \]  

with \( n \geq 1 \), real (not necessary natural) \( k > 1 \), and continuous functions \( p(x) \) and \( a_j(x) \), uniform estimates for positive solutions with the same domain ([1]) are obtained. For alternating-sign solutions the uniform estimates are obtained to the equation

\[ y^{(n)} + \sum_{j=0}^{n-1} a_j(x) y^{(j)} + p(x) |y|^k = 0 \]  

and for some special cases of equation (1).

**References**


\(^{1}\)The work was supported by RFBR, grant 08-01-00819
Nonanticipative strategies for \( n \)-person nonzero-sum differential games\(^*\)

Yurii Averboukh

Institute of Mathematics and Mechanics UrB RAS, Yekaterinburg, Russia, ayv@imm.uran.ru

We consider control system

\[
\dot{x} = f(t, x, u_1, \ldots, u_m), \quad t \in [t_0, \vartheta_0], \quad x \in \mathbb{R}^n, \quad u_i \in P_i.
\]

Here variable \( u_i \) denotes the control of the \( i \)-th player. The aim of the \( i \)-th player is to maximize his terminal payoff described by function \( \sigma_i \).

The game is considered in the framework of nonanticipative strategies. The nonanticipative strategies of the players \( 1, \ldots, i-1, i+1, \ldots, m \) is the map from the set of measure-valued controls of the \( i \)-th player to the set of all measure-valued joint controls of the players. Let \( \mathcal{M}^i[\alpha^i](t_*, x_*) \) be a bundle of motion generated by the the nonanticipative strategy \( \alpha^i \) of the players \( 1, \ldots, i-1, i+1, \ldots, m \).

The \( n \)-tuple of strategies \( \alpha^1, \ldots, \alpha^m \) is Nash equilibrium if for each \( i \)

\[
\max\{\sigma_i(x(\vartheta_0)) : x(\cdot) \in \mathcal{M}^i[\alpha^i](t_*, x_*)\} \\
\leq \min\{\sigma_i(y(\vartheta_0)) : y(\cdot) \in \mathcal{M}^1[\alpha^1](t_*, x_*) \cap \ldots \cap \mathcal{M}^m[\alpha^m](t_*, x_*)\}.
\]

Further we prove that the set of Nash equilibriums is fully described by the measure-valued joint controls of the players \( \eta \) such that

\[
\omega_i(t, x(t)) \leq \sigma_i(x(t, t_*, x_*, \eta)), \quad i = 1, m.
\]

Here \( \omega_i \) denotes the value of zero-sum differential game in which \( i \)-th wishes to maximize the payoff \( \sigma_i(x(\vartheta_0)) \), the goal of other players is opposite; \( x(\cdot, t_*, x_*, \eta) \) is a trajectory with initial position \((t_*, x_*)\) generated by the control \( \eta \). Also we obtain infinitesimal conditions on given multivalued map to be a Nash value of the game. These conditions are the generalizations of the approach based on system of Hamilton-Jacobi equations.

**Keywords.** Differential games, Nash equilibrium.

**AMS Subject Classifications.** 49N70, 91A10, 49L25.

\(^*\)The work is supported by RFBR (grant No 09-01-00436-a), Grant of President of Russian Federation (project MK-7320.2010.1), RAS Presidium Program of Fundamental Researches “Mathematical Theory of Control”.
Oscillation of solutions of linear discrete delayed equations

J. Baštinec J. Diblík Z. Šmarda

Brno Technological University, Brno, Czech Republic

AMS Subject Classification: 39A10, 39A11.

Keywords and Phrases: Discrete delayed equation, oscillating solution, positive solution, asymptotic behavior.

We consider the delayed second-order linear discrete equation

\[ \Delta x(n) = -p(n)x(n - 1) \]  

where \( n \in \mathbb{Z}_a^\infty := \{a, a + 1, \ldots \}, a \in \mathbb{N} \) is fixed, \( \Delta x(n) = x(n + 1) - x(n) \), \( p: \mathbb{Z}_a^\infty \to (0, \infty) \). It is known that if \( a \in \mathbb{N} \) is sufficiently large, \( q \in \mathbb{N} \) and \( p: \mathbb{Z}_a^\infty \to \mathbb{R}^+ \) satisfies

\[ p(n) \leq \frac{1}{4} + \frac{1}{(4n)^2} + \cdots + \frac{1}{(4n \ln n \ln_2 n \cdots \ln_q n)^2}. \]  

for every \( n \in \mathbb{Z}_a^\infty \) then there exists a positive integer \( a_1 \geq a \) and a solution \( x = x(n), n \in \mathbb{Z}_{a_1}^\infty \) of equation (1) such that \( x(n) > 0 \) holds for every \( n \in \mathbb{Z}_{a_1}^\infty \).

Our goal is to answer the open question whether all solutions of (1) are oscillating if inequality (2) is replaced by the opposite inequality

\[ p(n) \geq \frac{1}{4} + \frac{1}{(4n)^2} + \cdots + \frac{1}{(4n \ln n \ln_2 n \cdots \ln_{q-1} n)^2} + \frac{\kappa}{(4n \ln n \ln_2 n \cdots \ln_q n)^2} \]

assuming \( \kappa > 1 \) and \( n \) is sufficiently large.

Acknowledgement: The investigation was supported by the Grants 201/08/9469, 201/10/1032 of Czech Grant Agency, and by the Councils of Czech Government MSM 0021630503, MSM 0021630519 and MSM 0021630529.
Stability and Nonoscillation for Linear Differential Equations with Several Delays

Leonid Berezansky

Department of Mathematics, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

New explicit conditions of asymptotic and exponential stability and nonoscillation are obtained for the scalar and vector nonautonomous linear delay differential equation

\[ \dot{x}(t) + \sum_{k=1}^{m} a_k(t)x(h_k(t)) = 0 \]

with measurable delays and coefficients. These results are compared to known stability tests.

On reconstruction of structure of a linear system with time delay

Marina Blizorukova, Vyacheslav Maksimov
(Institute of Mathematics and Mechanics, Ekaterinburg, Ural branch, Academy of Science, Russia)

The problem of reconstruction of a structure of a linear system with delay

\[ \dot{x}(s) = Ax(s) + Bx(s - \tau), \quad s \in [0, T], \]

\[ x(\nu) = x_0(\nu), \quad \nu \in [-\tau, 0], \]

is considered. It is assumed that the system structure (i.e., matrices \( A \) and \( B \)) is unknown. It is only known that they belong to convex, bounded, and closed sets \( F_1 \subset R_1^{m \times q} \) and \( F_2 \subset R_2^{m \times q} \), respectively.

The algorithm is based on the combination of methods of the theory of guaranteed control and the method of smoothing functional (Tikhonov’s method), well-known in the theory of ill-posed problems.
Bounded Solutions of Impulsive Differential Systems.

Aleksandr Boichuk
Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv and Department of Mathematics, Faculty of Science, University of Zilina, 010 26 Zilina, Slovakia E-mail: boichuk@imath.kiev.ua
2000 MSC : 34A37,34A55,34B40

The problems of existence and structure of solutions bounded on the entire real axis of the linear and nonlinear impulsive differential systems are considered. Under assumption that the corresponding linear homogeneous system is exponentially dichotomous on the semi axes $\mathbb{R}_-$ and $\mathbb{R}_+$ and by using the theory of pseudo inverse matrices [1] we establish necessary and sufficient conditions for the indicated problem [2]. As an example of application of above-formulation results, we consider some applications in control theory and in the theory of bifurcations and branching theory of bounded solutions for linear and nonlinear impulsive differential systems with a small parameter. This research was supported by the Grants 1/0771/08, 1/0090/09 of the Grant Agency of Slovak Republic (VEGA) and project APVV-0700-07 of Slovak Research and Development Agency.

On the solvability of linear boundary value problems for functional differential equations 

Eugene Bravyi

(Perm State University, Perm, Russia)

Key Words: Functional differential equations, boundary value problems.
AMS(MOS) Subject Classification: 34K06, 34K10.

Functional differential equations (FDE) are relatively new mathematical objects. They arise in mathematical modelling in physics, biology, economics as a generalization of ordinary differential equations. The study of these more complex objects requires a combination of methods of ordinary differential equations and modern methods of functional analysis. Along with many other mathematicians significant contribution to the theory of FDE was made by Perm scientists N.V. Azbelev, V.P. Maksimov, L.F. Rahmatullina, M.E. Drakhlin and other members of the Perm Seminar on FDE under the direction of N.V. Azbelev. In Perm, the concept of Abstract FDE, new productive approaches to solving basic problems of FDE were proposed.

In this talk we present a new method to obtain the necessary and sufficient conditions for the solvability of boundary value problems for some families of FDE. The task of finding the best possible constants in solvability conditions always attracted mathematicians. For FDE this problem in many cases is not resolved yet. This is a promising area of research where there are many new results. The best constants in conditions for the solvability of the periodic problem for higher-order FDE were found only in 2009. The proposed new method is applied to various linear boundary value problems.

*Supported by Grant 10-01-96054-r-ural-a of The Russian Foundation for Basic Research

Use of Quasi-Normal Form to Examine Stability of Equilibria in a

Mathematical Model of BCG Treatment of Bladder Cancer

Svetlana Bunimovich and Yakov Golser
Tel-Aviv University and Ariel University Center, Israel

Abstract.

We present a mathematical model of immunotherapy for bladder cancer as an effect of BCG treatment. Understanding the dynamics of human hosts and tumors is of critical importance. This treatment exploits the host’s own immune system to boost a response that will enable the host to rid itself of the tumor. In this work, we analyze stability in a mathematical model for BCG treatment of bladder cancer based on the use of *quasi-normal form* and stability theory. These tools are employed in the critical cases, especially when analysis of the linearized system is insufficient. Our goal is to gain a deeper insight into the BCG treatment of bladder cancer, which is based on a mathematical model and biological considerations, and thereby to bring us one step closer to the design of a relevant clinical protocol.
Data mining techniques for asymptotic solutions to Vlasov-Maxwell equations

J. Chaskalovic

Ariel University Center & University Pierre and Marie Curie

Abstract

We propose a novel approach that consists in using data mining techniques to perform a sensitivity analysis of approximate models, and more generally, to scientific computing. As it is known, such techniques have proved to be efficient in other contexts which deal with huge data like biology, medicine, marketing, advertising and communications. In this work, we focus our presentation to a test case dedicated to an asymptotic paraxial approximation to model ultrarelativistic particles. Indeed, solving the time-dependent Vlasov Maxwell equations, which is the most complete mathematical model for collisionless plasma or non-collisional beams, can lead to very expensive computations especially in a three-dimensional domain. Despite some theoretical convergence results, it is not always easy to determine which terms to retain in the asymptotic expansion to get a sufficiently precise but not too expensive model. In other words, the asymptotic models are often difficult to compare directly one to the other. Then, our method directly deals with numerical results of simulations and try to understand what each order of the asymptotic expansion brings to the simulation results over what could be obtained by other lower-order or less accurate means? This new heuristic approach offers new potential applications to treat numerical solutions to mathematical models.

NUMERICAL-ANALYTIC METHOD INVESTIGATION OF SOME LINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

Valery Cherepennikov

Irkutsk State University
Irkutsk, K.Marks St.1, 664001, RUSSIA

AMS Class.: 34K06, 34K10.

The paper considers a boundary value problem for the following scalar functional differential equation of delay type

\[ \dot{x}(t) = a(t)x(t - 1) + b(t)x(t/q) + f(t), \quad t \in \mathbb{R}, \]

where the coefficients \( a(t), b(t) \) and \( f(t) \) are assumed to be polynomials.

For the purpose of investigation of boundary value problem (\(*\)) we have applied the method of polynomial quasisolutions [1, 2], which is based on the representation of the unknown function in the form of polynomial \( x(t) = \sum_{n=0}^{N} x_n t^n \). As a result of substitution of this function into equation (\(*\)), there appears a residual \( \Delta(t) = O(t^N) \), for which an exact analytical representation has been obtained. In turn, this allows one to find the unknown coefficients \( x_n \) and consequently the polynomial quasisolution \( x(t) \). A close relationship between the correctness of statement of the problem under scrutiny and the model equations with constant coefficients, whose structure of solution is defined by the roots of the characteristic quasipolynomial, is emphasized.

The results obtained are illustrated by examples.

References


Global solutions of semilinear first order partial functional differential equations with mixed conditions

W. Czernous

Institute of Mathematics, University of Gdask, Wit Stwosz Street 57, 80-952 Gdask, Poland
email: czernous@mat.ug.edu.pl, phone: +48 58 523 25 05, fax: +48 58 341 49 14

We consider the initial boundary value problem for a semi-linear partial functional differential equation of the first order,

$$
\partial_t z(t, x) + \partial_x z(t, x) f(t, x) = F(t, x, z(t, x) \alpha(t, x)),
$$

Using the method of characteristics and the sequence of successive approximations, we prove the global existence, uniqueness and continuous dependence on data of classical $C^1$ solutions of the problem. This approach covers equations with deviating variables, namely the above one, with right hand side

$$
F(t, x, z(\alpha_0(t, x), \alpha_1(t, x), \ldots, \alpha_n(t, x)),$$

as well as integro-differential equations, where admissible right-hand sides are

$$
\bar{F} \left( t, x, \int_D z (\alpha_0(t, x) + s, \alpha_1(t, x) + y_1, \ldots, \alpha_n(t, x) + y_n) ds dy \right).
$$

The Gâteaux differential of the solution operator with respect to the initial-boundary data, or with respect to the pair: the data and the right-hand side, is found.

AMS Subject Classifications: 35R10.

Key words: classical solutions, global existence, characteristics, differentiability with respect to data.

Generalized Exchange in Magnetic Resonance

Dan Gamliel

Ariel University Center, Ariel, Israel

In nuclear magnetic resonance (NMR) experiments one obtains a spectrum or "lineshape", which is the superposition of different spectral lines. Interactions between spins cause a spectral line to split into a set of lines. Motion of spins affects the lineshape in a characteristic manner. In particular, thus is true for site exchange between different spins. One can characterize the transition from slow motion (exchange) to fast motion (exchange) by a gradual change of the lineshape from that of the static system to the average over the dynamic system.

Standard equations are used to calculate such exchange processes, assumed to be instantaneous. We are interested in the generalization to a process of exchange with a delay. We compare two approximate descriptions of the system. In one model the transition state is assumed to have negligible influence on the lineshape, and the process is simulated with a time delay. In the other model the transition state is treated on an equal footing with the ordinary states between which the exchange process is being executed, and then no delay is needed. The results of the two models are compared, and the domain of validity of each model is discussed.
Positive solutions of functional differential delayed equations

J. Diblik, Z. Svoboda, Z. Šmarda, A. Ryvolová

Brno Technological University, Brno, Czech Republic

AMS Subject Classification: 34K20, 34K25, 39A10, 39A11.

Keywords and Phrases: Functional delayed equation, positive solution, asymptotic behavior.

Let $C([a,b], \mathbb{R}^n)$, where $a < b$, $\mathbb{R} = (-\infty, +\infty)$, be the Banach space of continuous functions mapping the interval $[a,b]$ into $\mathbb{R}^n$. If $a = -r < 0, b = 0$, we denote this space as $C$. Let

$$\dot{y}(t) = f(t, y_t)$$

be a system of retarded functional differential equations where $f : \Omega \mapsto \mathbb{R}^n$ is a quasibounded continuous mapping satisfying a local Lipschitz condition with respect to the second argument and $\Omega$ is an open subset in $\mathbb{R} \times C$. Some criteria of existence of positive solutions of (1) will be presented. As a particular case a first-order differential equation containing delay

$$\dot{y}(t) = -p(t)y^\alpha(t - r)$$

with $p : (t_0, \infty) \to (0, \infty)$ and $\alpha \geq 1$ will be considered. A criterion of the existence of positive solutions (for $t \to \infty$) will be demonstrated. Connections between the nonlinear case ($\alpha \neq 1$) and the linear case ($\alpha = 1$) will be discussed. Some analogue with discrete equations will be mentioned as well.

Acknowledgement: The investigation was supported by the grants 201/08/9469 and P201/10/1032 of Czech Grant Agency, by the Councils of Czech Government MSM 0021630503, MSM 0021630519 and MSM 0021630529 and by the project FEKT-S-10-3 of Faculty of Electrical Engineering and Communication, Brno University of Technology.

On the unique solvability of a non-linear non-local boundary value problem for systems of non-linear functional differential equations

N. Dilna, A. Rontó

Mathematical Institute of the Slovak Academy of Sciences, Bratislava
Institute of Mathematics of the National Academy of Sciences of Ukraine, Kiev
Institute of Mathematics of the Academy of Sciences of the Czech Republic, Brno

This is a joint work with A. Ronto. We study the non-local boundary value problem for the system of functional differential equations

$$u'_k(t) = (f_k)u(t), \quad t \in [a,b], \quad k = 1, 2, \ldots, n,$$

$$u_k(a) = \varphi_k(u), \quad k = 1, 2, \ldots, n,$$

where $f_k : D([a,b], \mathbb{R}^n) \to L_1([a,b], \mathbb{R}), k = 1, 2, \ldots, n$, are, generally speaking, non-linear operators and $\varphi_k : D([a,b], \mathbb{R}^n) \to \mathbb{R}, k = 1, 2, \ldots, n$, are non-linear functionals defined on the space $D([a,b], \mathbb{R}^n)$ of vector functions with absolutely continuous components. General conditions sufficient for the unique solvability of problem (1), (2) are obtained [1].

Key words: Non-linear boundary value problem, functional differential equation, non-local condition, unique solvability

2000 Mathematics Subject Classification: 34K10, 34K38

References


*Research supported in part by the Grant VEGA-SAV 2/7140/27.
Maximum Principles for One of the Components of Solution Vector for System of Functional Differential Equations

Alexander Domoshnitsky
Ariel University Center, Ariel, Israel, adom@ariel.ac.il

The method to compare only one component of the solution vector of linear functional differential systems, which does not require heavy sign restrictions on their coefficients, is proposed in this talk. Necessary and sufficient conditions of the positivity of elements in a corresponding row of Green’s matrix are obtained in the form of theorems about differential inequalities. The main idea of our approach is to construct a first order functional differential equation for the \( n \)th component of the solution vector and then to use assertions about positivity of its Green’s functions. This demonstrates the importance to study scalar equations written in a general operator form, where only properties of the operators and not their forms are assumed. It should be also noted that the sufficient conditions, obtained in this talk, cannot be improved in a corresponding sense and in many cases does not require any smallness of the interval \([0, \omega]\), where the system is considered.

On Maximum Principles for Neutral Functional Differential Equations of the First order
Alexander Domoshnitsky, Abraham Maghakyan and Roman Shklyar
Ariel University Center of Samaria, Ariel, Israel

In this paper we obtain the maximum principles for the first order neutral functional differential equation

\[
(Mx)(t) \equiv x'(t) - (Sx')(t) - (Ax)(t) + (Bx)(t) = f(t), \quad t \in [0, \omega],
\]
where \( A : C_{[0,\omega]} \to L^\infty_{[0,\omega]} \), \( B : C_{[0,\omega]} \to L^\infty_{[0,\omega]} \) and \( S : L^\infty_{[0,\omega]} \to L^\infty_{[0,\omega]} \) are linear continuous operators, \( A \) and \( B \) are positive operators, \( C_{[0,\omega]} \) is the space of continuous functions and \( L^\infty_{[0,\omega]} \) is the space of essentially bounded functions defined on \([0, \omega]\). New tests on positivity of the Cauchy function and its derivative are proposed. Results on existence and uniqueness of solutions for various boundary value problems are obtained on the basis of the maximum principles.

Boundary value problems for first order functional differential equations with non–Volterra’s operators

A. Domoshnitsky, R. Hakl, B. Puža

Efficient conditions for the unique solvability of the boundary value problem for linear functional differential equations of the form

\[
u'(t) = \ell(u)(t) + q(t), \quad h(u) = c
\]
are established. Here, \( \ell : C([a, b]; R) \to L([a, b]; R) \) and \( h : C([a, b]; R) \to R \) are linear bounded operators, \( q \in L([a, b]; R) \), and \( c \in R \). The results are based on the conditions guaranteeing that the solution set of the corresponding homogeneous equation is one–dimensional, generated by a positive monotone function.

Estimates of Zones where All Solutions of Functional Equations Change their Signs

Alexander Domoshnitsky and Roman Yavich
Ariel University Center of Samaria, Ariel, Israel

We obtain assertions about possible sizes of zones of positivity of solutions to functional equations. Our approach is based on theorems about estimates of the spectral radius. Simple coefficient tests are proposed.
Stability results for FDEs with variable impulses via Generalized ODEs.

Marcia Federson
Sao Paulo University, Sao Paulo, Brazil

We consider a class of functional differential equations with variable impulses and we establish new stability results which encompass previous ones. In order to obtain our results, we discuss the variational stability and variational asymptotic stability of the zero solution of a class of generalized ordinary differential equations where our impulsive functional differential equations can be embedded and we apply that theory to obtain our results, also using Lyapunov functionals.

ON SOLVABILITY OF SOBOLEV TYPE EQUATION WITH DELAY

Vladimir E. Fedorov, Ekaterina A. Omelchenko
(Chelyabinsk State University, Chelyabinsk, Russia)

Let $U$ and $\mathfrak{F}$ be Banach spaces, operators $L : U \to \mathfrak{F}, \ker L \neq \{0\},$ and $\Phi : C([-r,0];U) \to \mathfrak{F}$ be linear and continuous, operator $M : \text{dom} M \to \mathfrak{F}$ be densely defined in $U,$ linear, closed and strongly $(L,p)$-radial [1]. Then $\mathfrak{F} = \mathfrak{F}^0 \oplus \mathfrak{F}^1,$ $\mathfrak{F}^0 = \mathfrak{F}^0 \oplus \mathfrak{F}^1$ and there exist homeomorphic operator $L_1 : \mathfrak{F}^1 \to \mathfrak{F}^1$ and a generator $L_1^{-1}M_1$ of $C_0$-semigroup on $\mathfrak{F}^1.$

Let $P (Q)$ is a projector on $\mathfrak{F}^1 (\mathfrak{F}^0)$ along $\mathfrak{F}^0 (\mathfrak{F}^1).$ Denote by $T$ the operator $Tz = z'$ defined on the domain $\text{dom} T = \{ z \in C^1([-r,0];\mathfrak{F}^1) : z(0) \in \text{dom} L_1^{-1}M_1, z'(0) = L_1^{-1}M_1z(0) + L_1^{-1}Q\Phi z \}.$

**Theorem.** Let operator $M$ be strongly $(L,p)$-radial, $\Phi \in \mathcal{L}(C([-r,0];U);\mathfrak{F}),$ $\text{im} \Phi \subset \mathfrak{F}^1,$ $Ph \in \text{dom} T,$ $(I - P)h \in C([-r,0];\mathfrak{F}^0),$ $(I - P)b(0) = 0.$ Then there exists a unique solution $u \in C([-r, +\infty);\mathfrak{F}) \cap C^1([0, +\infty);\mathfrak{F})$ of the problem

$$u(t) = b(t), \quad t \in [-r,0]; \quad L\dot{u}(t) = Mu(t) + \Phi u_t, \quad t \in [0, +\infty).$$

Here $u_t \in C([-r,0];\mathfrak{F}),$ $u_t(s) = u(t+s), s \in [-r,0].$

**Bibliography**


The work is supported by Russian Foundation for Basic Research, No. 10-01-96007-r_ural_a.

Keywords: Sobolev type equation, equation with delay, semigroup of operators.

AMS Subject Classification: 34G10, 47D06.
On the decay for solutions of the weighted wave equation in unbounded domains
Filinovskiy A.V. (Moscow)
flnv@yandex.ru

MR 35L20, 47F05
Keywords: weighted wave equation, unbounded domain; initial-boundary value problem; stabilization; elliptic operator; discrete spectra; continuous spectra

Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$, be an unbounded domain whose closure does not contain the origin. In the semi-infinite cylinder $\{ t > 0 \} \times \Omega$ we consider the initial-boundary value problem for the equation $u_{tt} + Lu = 0$, $u|_{t=0} = f$, $u_t|_{t=0} = g$, $u|_{\partial \Omega} = 0$, where $L = -r^s\Delta$, $r = |x|$, $s \geq 0$, is the weighted Laplace operator. For different values of parameter $s$ we study the asymptotic behavior at $t \to \infty$ of solutions for this problem under the assumption that the boundary $\partial \Omega$ satisfies a star-shapedness condition with respect to the origin. Our special interests also are spectral properties of the elliptic operator $L$ generated by formally non-self-adjoint differential expression. We shall treat this differential operator in the Hilbert space $L_{2,s}(\Omega)$ with norm $\|u\|^2_{L_{2,s}(\Omega)} = \int_{\Omega} r^{-s}|u|^2 \, dx$, therefore, $L$ is a positive self-adjoint operator in $L_{2,s}(\Omega)$ that is an operator of the Dirichlet problem for the weighted Laplace operator. For different values of $s$ we investigate the location of spectrum $\sigma(L)$ on the real axis and density of spectrum on some sets. We prove that for $0 \leq s \leq 2$ the $\sigma(L)$ is continuous and for $s > 2$ the $\sigma(L)$ is discrete (compare [1], [2]). Further we give an estimate to the rate of condensation of discrete spectrum under the transition to continuous. For $0 \leq s < 2$ the absolute continuity of $\sigma(L)$ is proved and the estimates for resolvent of $L$ are established.

References

Time uncertainty and precision of differential equations

E.A. Galperin,
University of Montreal, Montreal, Canada

A measured point-wise value of time $t$ or some other quantity $z(t)$ depending on time, when transmitted by a physical process, relates to an instant which, at the moment of reception, is already in the past. If transmission is carried over a short length with the speed of light, its time duration $d > 0$ is very small, so transmitted $z(t)$ is considered at reception as current value despite that, in fact, it is already past, the current value being $z(t+d)$ where $d > 0$ is unknown and depends on a finite speed of information transmittal. It will be demonstrated that a distance locator based on radar measurements can produce errors of $30,000$ km with $0.1$ sec delay in its information transmission through a computer system.

The consideration of $z(t)$ instead of $z(t+d)$ creates time uncertainty which affects physical experiments, real time computations and process evolution. This has important implications for mathematical description of physical processes which cannot be accurately described by ODEs or PDEs, but require DDEs or FDEs.

This issue, as well as the necessity to include the left higher order derivatives in the forces at right-hand sides with possible advance terms therein will be discussed. As an example, a possible explanation of the recent accident of the Air France flight Rio de Janeiro - Paris, and of the visible instability of the super-jumbo jet A-380 at landing in Heathrow airport in London will be presented.

Investigation of Integro-Differential Equations with distributed delay by reduction method

Yakov Golts er
Department of Mathematics and Computer Sciences,
Ariel University Center of Samaria, Israel

Abstract

Our goal is to study the Integro-Differential equations in $\mathbb{R}^n$ with distributed delay (IDEDD)

$$\frac{dx}{dt} = X(t,x, \int_{-\infty}^{\infty} K(t,s) f(s,x(s))ds),$$

where the kernel has the form

$$K(t,s) = \sum_{j=1}^{\infty} F_j(t) R_j(s).$$

We establish a connection between system IDEDD and countable system of ODE. Such a reduction allows use of results obtained earlier for the countable systems of ODE in study of integro-differential equations. In the paper we will discuss problems related to applications of the reduction principle in the theory of stability.
The system of complement and many other biochemical systems (blood coagulation, extracellular matrix degradation, granzyme-B induced apoptosis, and many more) are modeled as large systems of first order non-linear ODE with numerical and functional parameters. Using the system of complement as an example, we discuss some of the fundamental features of the behavior of these dynamical systems such as stability of the background state, low sensitivity to the change in initial conditions and kinetic constants, and presence of a considerable lag phase. The main problem is what structural properties of the system of ODE bring about the observed patterns of behavior of the underlying biochemical system.

THE ASYMPTOTIC EQUIVALENCE OF DIFFERENTIAL EQUATIONS WITH IMPULSE ACTION

V. Kravets and V. Mogilova
Ahrotehnolohichnyy Tavria State University of Melitopol (Ukraine), National Technical University of Ukraine 'Kyiv Polytechnic Institute'
e-mail: v_i_kravets@list.ru

We consider the following system of differential equations with impulse action

\[ \dot{x} = f(t, x), t \neq \tau_k(x), \quad \Delta x|_{t=\tau_k(x)} = h_k(x), \]  
(1)

and the system of ordinary differential equations

\[ \dot{y} = g(t, y). \]  
(2)

Definition 1. We say that systems (1) and (2) are asymptotic equivalence if there is isomorphism between their solutions when

\[ \lim_{t \to \infty} |x(t) - y(t)| = 0. \]  
(3)

We study the conditions when will be the asymptotic equivalence.

Theorem 1. Let the following conditions hold true:

1) \( f(t, x) \) – satisfy the Lipschitz condition on \( x \in \mathbb{R}^n \) with constant \( L \), \( f(t, 0) \) - is bounded;

2) the series \( \sum_{k=1}^{\infty} \sup_{x \in \mathbb{R}^n} ||h_k(x)||e^{L\tau_{k+1}(x)} \) – is converges.

Then the system

\[ \dot{y} = f(t, y) \]

is asymptotic equivalence to the system (1).
The estimates of converge of solutions linear stochastic equations of neutral type

Dzhalladova I., Khusainov D.J.

(V. Getman Kyiv National Economical University, T.Shevchenko Kyiv National University, Ukraine, Kyiv)

Consider the linear stochastic function-differential equations of neutral type

1. Statement of problem

On probability space considering of linear stochastic function-differential equations of neutral type

\[ d[x(t) - cx(t-\tau)] = [a_0x(t) + a_1x(t-\tau)]dt + [b_0x(t) + b_1x(t-\tau)]dw(t), x(t) \in (R^1). \]  

(1)

where \(a_0, a_1, b_0, b_1, c\) – constant values, \(\tau > 0\)–constant delay, \(w(t)\) – scalar standart Winner process:

\[ M(dw(t)) = 0, M(dw(t))^2 = dt, M(dw(t)dw(t_1), t \neq t_1) = 0. \]  

(2)

In investigating problems, we use Lyapunov-Krasovsky functional in next form

\[ V[x(t), t] = e^{\gamma t}[x(t) - cx(t - \tau)]^2 + g \int_{-\tau}^{0} e^{\beta s} x^2(s) ds, h > 0, g > 0, \]  

(3)

We denote

\[ S[g, \beta, \gamma] = \begin{bmatrix} A - \gamma - g & B + \gamma c \\ B + \gamma c & ge^{-\beta \tau} - \gamma c^2 + C \end{bmatrix} \]

\[ A = -2a_0 - b_0^2, \quad B = -a_1 + a_0c - b_0b_1, \quad C = 2a_1c - b_1^2 \]

\[ \lambda_{\text{min}}(S[g, \beta, \gamma]) \] –minimal eigenvalues of matrix \(S[g, \beta, \gamma]\).

2. Main results.

Theorem. Let exist constant \(\beta > 0, g > 0, \gamma > 0, \beta > \gamma\) such that holds next equations

\[ \Delta_1 = A - \gamma - g > 0, \Delta_2 = (A - \gamma - g)(ge^{-\beta \tau} - \gamma c^2 + C) - (B + \gamma c)^2 > 0 \]  

(4)

Then zero solutions is exponential \((\gamma, \beta)\) - integral stability in mean square and for arbitrary solutions \(x(t)\) rough next estimate of convergence

\[ M\{\|x(t)\|^2_{\tau, \beta}\} \leq \sqrt{\frac{1 + c^2}{g} (|x(0)| + |x(-\tau)|) + \|x(0)\|_{\tau, \beta} e^{-\theta(\beta, \gamma)t}} \]

\[ t \geq 0, \theta(\beta, \gamma) = \min\{\beta, \frac{\lambda_{\text{min}}(S[g, \beta, \gamma])}{1 + c^2} + \gamma\} \]  

(5)

Similarly results we obtain for the system equations.
Abstract

The linear partial delay differential equations are considered:

\[ u_t(x, t) = a_1^2 u_{xx}(x, t) + a_2^2 u_{xx}(x, t - \tau) + c_1 u(x, t) + c_2 u(x, t - \tau) + f(x, t), \]

It is defined within \( 0 \leq x \leq l, t \geq 0 \). The first boundary problem is considered. The initial conditions

\[ u(x, t) = \varphi(x, t), 0 \leq x \leq l, -\tau \leq t \leq 0, \]

and boundary conditions

\[ u(0, t) = \mu_1(t), u(l, t) = \mu_2(t), t \geq -\tau. \]

And "consistency" conditions are satisfied

\[ \varphi(0, t) = \mu_1(t), \varphi(l, t) = \mu_2(t), -\tau \leq t \leq 0. \]

The Fourier method is used for solving. For the spatial variables the Sturm-Liouville problem is solved. For the time variables the sequence of the delay equations is considered:

\[ \dot{T}_n(t) = \left[ c_1 - \left( \frac{\pi n}{l} a_1 \right)^2 \right] T_n(t) + \left[ c_2 - \left( \frac{\pi n}{l} a_2 \right)^2 \right] T_n(t - \tau), n = 1, 2, 3, \ldots \]

with initial conditions received after decomposition of corresponding initial conditions using the Fourier series. Using the special function named retarded exponential function, the solutions for each of the equations for the time variables are received in integral form. The solution for the first boundary problem is obtained as the series expansion.
Consider the differential equation

\[ u^{(n)}(t) + p(t)|u(\sigma(t))|^{\mu(t)} \text{sign}(\sigma(t)) = 0, \tag{1} \]

where \( p \in L_{\text{loc}}(\mathbb{R}_+; \mathbb{R}), \quad \sigma \in C(\mathbb{R}_+; \mathbb{R}) \) and \( \lim_{t \to +\infty} \sigma(t) = +\infty, \quad \mu \in C(\mathbb{R}_+; (0, +\infty)) \).

**Definition 1.** We say that the equation (1) is almost linear, if the condition

\[ \lim_{t \to +\infty} \mu(t) = 1 \tag{2} \]

is fulfilled.

**Definition 2.** We say that the equation (1) is essentially nonlinear, if at least one of the following conditions

\[ \liminf_{t \to +\infty} \mu(t) \neq 1 \quad \text{or} \quad \limsup_{t \to +\infty} \mu(t) \neq 1 \tag{3} \]

is fulfilled.

We study oscillatory properties of solutions of equation (1) for the following cases (2) or (3).
On Khavinson’s extremal problem for harmonic functions

Gershon Kresin
Department of Computer Science and Mathematics
Ariel University Center of Samaria
44837 Ariel, Israel
kresin@ariel.ac.il

Abstract. Sharp estimates for harmonic functions are important in problems relating electrostatics as well as hydrodynamics of ideal fluid, elasticity and hydrodynamics of the viscous incompressible fluid. The generalization of D. Khavinson’s extremal problem for harmonic functions is described as follows. Let \( \mathcal{H} \) be a class of harmonic functions in the ball \( B = \{ x \in \mathbb{R}^n : r = |x| < 1 \} \), and let \( \Phi(u) \) be a positively homogeneous functional defined on boundary values of functions in \( \mathcal{H} \) such that

\[
|\langle \nabla u(x), \ell \rangle| \leq C(x)\Phi(u)
\]

for all \( |\ell| = 1 \) and \( u \in \mathcal{H} \). One is looking for directions \( \ell \) for which the sharp constant \( K_{\Phi}(x; \ell) \) in

\[
|\langle \nabla u(x), \ell \rangle| \leq K_{\Phi}(x; \ell)\Phi(u), \quad u \in \mathcal{H},
\]

attains its minimal and maximal values. A similar problem can be considered in other domains (for example, in the half-space).

We discuss the generalized Khavinson’s problem in the multidimensional ball as well as in the half-space under the assumption that the function’s boundary values belong to \( L^p \). The talk is based on the results of the joint work with V. Maz’ya.
Existence and Asymptotic Behavior of Positive Solutions of a Functional Differential Equation

Mária Kúdelčíková

Author’s address: Department of Mathematics, Faculty of Science, University of Žilina, Žilina, Slovak Republic

E-mail address: maria.kudelcikova@fpv.uniza.sk

AMS Subject Classification: 34K15, 34K25

Key words: Delayed differential equation, retract method, positive solutions, asymptotic behavior of solutions.

Abstract

The functional differential equation
\[ \dot{y}(t) = -f(t, y_t) \]  
(1)

is considered for \( t \to +\infty \), where \( f: \Omega \to \mathbb{R} \) is a continuous quasi-bounded functional that satisfies a local Lipschitz condition with respect to the second argument and \( \Omega \) is an open subset in \( \mathbb{R} \times C([-r, 0], \mathbb{R}) \).

The existence of two classes of positive solutions of equation (1) which are asymptotically different is proved using the retract method combined with Razumikhin’s technique. It is used the supposition that the right hand side of the equation (1) can be estimated as follows:

\[ C_A(t) y_t(-r) \leq f(t, y_t) \leq C_B(t) y_t(-r), \]  
(2)

where \( (t, y_t) \in \Omega \) and \( C_A, C_B \) are positive continuous functions on \( [t_0, \infty) \), \( t_0 \in \mathbb{R} \) satisfying

\[ 0 < C_A(t) \leq C_B(t) \leq \frac{1}{r \varepsilon} \quad \text{and} \quad \int_{t_0}^{\infty} C_B(s) \, ds < \infty. \]

With the auxiliary linear equations, which are constructed using (2), inequalities for both types of positive solutions are given as well. \(^1\)

\(^1\)Acknowledgment. This research was supported by the Grant No 1/0090/09 of the Grant Agency of Slovak Republic (VEGA) and by the project APVV-0700-07 of Slovak Research and Development Agency.

Dynamical approach to ballistic transport in grapheme

Meir Lewkowitz

Ariel University Center of Samaria, Ariel, Israel

Abstract: The process of the coherent creation of particle–hole excitations by an electric field in graphene is quantitatively described beyond linear response. We calculate the evolution of the current density and the number of pairs in the ballistic regime using the tight binding model. While for small electric fields the I–V curve is linear characterized by the universal minimal resistivity \( \sigma = \pi/2(e^2/h) \), for larger fields, after a certain time interval, the linear regime crosses over to a quadratic one and finally at larger times Bloch oscillations set in.
On the method of control models in problems of dynamical reconstruction of characteristics of functional-differential systems

V. I. MAKSIMOV
Institute of Mathematics and Mechanics,
Ural Branch, Acad. Sci. of Russia,
S. Kovalevskaia str. 16, 620219. Ekaterinburg, Russia

The problems of reconstruction of unknown characteristics of dynamical systems described by different classes of functional-differential equations are under investigation. Solving algorithms, which are stable with respect to informational noises and computational errors and operate in real time mode, are designed. These algorithms are based on the method of control models from the theory of ill-posed problems and on the well-known in the theory of positional differential games control principle with a model. Inaccurate measurements of current phase states of systems play the role of input data for the algorithms, which provide some values approximating real (but unknown) characteristics. The basic elements of the algorithms are represented by stabilization procedures (functioning by the feedback principle) for appropriate Lyapunov functionals. The main goal of this presentation is to show how a priori information influences choosing a concrete algorithm.

Oscillatory and asymptotic properties of solutions of nonlinear neutral difference equations

MAŁGORZATA MIGDA and JAROSŁAW WERBOWSKI

Poznań University of Technology
Institute of Mathematics
Piotrowo 3A, 60-965 Poznań, Poland
email: malgorzata.migda@put.poznan.pl
email: jaroslaw.werbowski@put.poznan.pl

Keywords: neutral difference equation, asymptotic behavior, oscillation.

AMS Mathematics Subject Classification: 39A10

We study asymptotic properties and oscillation of solutions of difference equations of the form

\[ \Delta^m(x_n + p_n x_{n-\tau}) + f(n, x_n, x_{n-\sigma}) = 0 \]

where \( m \geq 2 \), \((p_n), (h_n)\) are sequences of real numbers, \( \tau \) and \( \sigma \) are nonnegative integer, \( f : N \times R \times R \longrightarrow R \).

We examine the case when the sequence \((p_n)\) is of constant sign and when \((p_n)\) is an oscillatory sequence as well.
Low-dimensional models for pattern forming systems under feedback control

Y. Kanevsky, A.A. Nepomnyashchy

Dept. of Mathematics, Technion - Israel Institute of Technology, 
Haifa, Israel

We investigate the influence of a nonlocal feedback control on the subcritically unstable pattern-forming systems. In some cases, the control allows to stabilize solitary waves which are characterized by a small number of variables. In that case, by means of a variational approach, the problem can be reduced approximately to a system of ordinary differential equations with nonlocal terms and with or without temporal delay. The approach is applied to some generic equations of the pattern formation theory, specifically to the subcritical Ginzburg-Landau equation coupled with a diffusion equation for a Goldstone mode. Transitions between different dynamical regimes are studied analytically and numerically.

THE THIN FILM EQUATION WITH BACKWARDS SECOND ORDER DIFFUSION

A. NOVICK-COHEN$^1$, A. SHISHKOV$^2$
(HAIFA$^1$, ISRAEL$^1$, DONESK$^2$, UKRAINE$^2$)

We treat the thin film equation with lower order ”backwards” diffusion which can describe, for example, structure formation in biofilms and the evolution of thin viscous films in the presence of gravity and thermo-capillary effects, focusing in particular on the equation

$$u_t + \{u^n(u_{xxx} + \nu u^{m-n}u_x - Au^{M-n}u_x)\}_x = 0,$$

where $\nu = \pm 1$, $n > 0$, $M > m$, $A \geq 0$. Global existence of weak nonnegative solutions is proven when $m - n > -2$, and $A > 0$ or $\nu = -1$, and when $-2 < m - n < 2$ and $A = 0$, $\nu = 1$. From the weak solutions, we get strong entropy solutions under the additional constraint that $m > n - 3/2$ if $\nu = 1$. A local energy estimate is obtained when $2 \leq n < 3$ under some additional restrictions. Finite speed of propagation is proven for the case of ”strong slippage,” $0 < n < 2$, when $m > n/2$ and $\nu = 1$, based on local entropy estimates, and for the case of ”weak slippage,” $2 \leq n < 3$, when $m > n/2$, based on local entropy and energy estimates.
The results described in this lecture are joint work with John Mallet-Paret.

Consider the linear differential-delay equation (*)

\[ x'(t) = -a(t)x(t) - b(t)x(t-1), \]

where \( a(t) \) and \( b(t) \) are given continuous, real-valued functions for \( t \geq d \). Equation (*) may arise by linearizing the equation (**) \( x'(t) = f(x(t), x(t-1)) \)

about a given solution \( x_0(t) \) of (**).

Define \( X := C([-1,0]) \), the Banach space of continuous, real-valued functions \( g \) from \([-1,0]\) to the reals. If \( g \) is an element of \( X \), there is a unique function \( x(t;g) \) which solves (*) for \( t \geq d \) and satisfies \( x(d+s) = g(s) \) for \(-1 \leq s \leq 0\). For \( c > 0 \), define a bounded linear map \( A := U(d+c,d) : X \to X \)

\[ (A(g))(s) := x(d+c+s;g) \]

for \(-1 \leq s \leq 0\). The \( m \)-fold (injective) tensor product of \( X \) with itself is a Banach space which can be canonically identified with \( C(M) \), where \( M \) is the \( m \)-fold Cartesian product of \([-1,0]\) with itself. The \( m \)-fold exterior product of \( X \) with itself, \( Y_m \), is a closed linear subspace of \( X_m \). \( Y_m \) contains a closed cone \( T_m \) consisting of functions \( f \) in \( Y_m \) such that

\[ f(s_1, s_2, ..., s_m) \geq 0 \]

whenever \(-1 \leq s_1 \leq s_2 \leq ... \leq s_m \leq 0\). If \((-1)^m (b(t)) > 0 \) for all \( t \geq d \), we prove that the \( m \)-fold exterior product of \( A \) with itself \( T_m \) into \( T_m \) and has positive spectral radius.

When combined with known results about positive linear operators and the spectrum of tensor products of linear operators, this result suggests a variety of new directions of research and yields new results even when \( a(t) \) and \( b(t) \) are constant.
In this talk we will present a Perron type theorem about the decay rates of the solutions in the neighborhood of the zero equilibrium of nonlinear delay differential equations. We will show that the Liapunov exponents of the nonnegative solutions tending to zero equilibrium are related to the nonpositive (real) eigenvalues of the linearization about the zero equilibrium. The proof is based on the discrete counterpart of the result which may be viewed as a generalization of the weak form of the Perron-Frobenius theorem.

**Keywords:** Delay differential equation; Nonnegative solution; Linearization; Perron’s theorem

**AMS Classification:** 34K25; 39A10

**About one problem in theory of singular functional differential equations.**

I. Plaksina
Perm State Polytechnic University, Perm, Russia

This article discusses some problems of functional differential equations with singularity of special type. These are conditions of Fredholm property and of solvability.

Key words: functional differential equations, singular equations, Fredholm property.

AMS (MOS) Subject Classification 34K06, 34K26, 47A53.

**Modeling as a way of research of the functional-differential equations**

M.Plaksin and V.Plaksina
State University Higher School of Economics (Perm Branch) and Perm State University, Perm, Russia,

In the report the application of technology of modeling for research of the functional-differential equations is considered. A functional-differential equation is replaced by model - the ordinary differential equation. The properties proved for the model equation, are transferred on the initial equation.
Simplification of complex systems is a widely accepted way of their modeling, the inevitable drawback of which is the "resolution reduction", i.e. the loss of some information inherent to the real-world processes. A "naive" modeling approach, which unfortunately dominates in the modern systems biology, simply disregards this drawback. In this talk we focus on effects of the "resolution reduction" in a rigorous way.

In particular, we will address some open mathematical problems which link two basic simplification paradigms in systems biology: the Boolean-like (BL) formalism and the power-law (PL) formalism.

The BL formalism is widely used for describing gene regulatory networks. It consists in replacing smooth yet inconvenient steep sigmoid nonlinearities with simple step functions. Usually it is assumed that the underlying functions should be affine. The PL formalism means that unknown or numerically obtained relationship can be described by sums of power monomials.

In the talk we demonstrate that the conventional affine models of gene regulatory networks are only sufficient at coarser levels of resolution, while at higher levels one needs to combine the PL formalism with the BL formalism.
On the local Cauchy problem for first order partial differential functional equations.

Elżbieta Pużniakowska - Galuch
Institute of Mathematics University of Gdańsk

We consider the functional differential equation

$$\partial_t z(t, x) = f(t, x, z(t, x), z_\varphi(t, x), \partial_x z(t, x))$$

with the initial condition

$$z(t, x) = (t, x) \text{ on } E_0$$

where $x = (x_1, \ldots, x_n)$, $\partial_x z = (\partial_{x_1} z, \ldots, \partial_{x_n} z)$, and $E_0 = [-b_0, 0] \times [-b, b]$, $E_0 \subset R^{1+n}$.

A theorem on the existence of weak solutions defined on the Haar pyramid is proved. The initial problem is transformed into a system of functional integral equations for an unknown function and for their partial derivatives with respect to spatial variables. A method of bicharacteristics and integral inequalities are applied. Differential equation with deviated variables and differential integral equation can be obtained from a general theory by specializing given operators.

Homoclinic solutions of singular non-autonomous differential equations

Irena Rachůnková
Palacky University, Olomouc, Czech Republic

Boundary value problems of the form

$$(p(t)u'(t))' = p(t)f(u(t)), \quad u'(0) = 0, \quad \lim_{t \to \infty} u(t) = c \geq 0 \quad (1)$$

are discussed. The function $p : [0, \infty) \to [0, \infty)$ has a continuous first derivative which is positive on $(0, \infty)$ and $p(0) = 0$. Therefore the equation in (1) has a singularity at $t = 0$. The function $f$ is supposed to be locally Lipschitz continuous on the real line and it has at least two zeros $0$ and $L$, where $0 < L$.

We provide conditions which guarantee the existence of solutions of problem (1) in $C^1[0, \infty) \cap C^2(0, \infty)$. Negative starting values of solutions converging to $c = 0$ or $c = L$ for $t$ going to $\infty$ are specified. By an extension of (1) to $R$, the above solutions are homoclinic ones and their global behaviour, in particular negativity, monotonicity or number of zeros, is characterized here.

Problems of this type arise for example in hydrodynamics, in population genetics, in the homogenous nucleation theory, or in the nonlinear field theory.
Consider the following system of impulsive differential equations in Banach space \( X \times Y \):

\[
\begin{align*}
\frac{dx}{dt} &= A(t)x + f(t, x, y), \\
\frac{dy}{dt} &= B(t)y + g(t, x, y), \\
\Delta x\big|_{t = \tau_i} &= x(\tau_i + 0) - x(\tau_i - 0) \\
\Delta y\big|_{t = \tau_i} &= y(\tau_i + 0) - y(\tau_i - 0) \\
\Delta x\big|_{t = \tau_i} &= C_i x(\tau_i - 0) + p_i(x(\tau_i - 0), y(\tau_i - 0)), \\
\Delta y\big|_{t = \tau_i} &= D_i y(\tau_i - 0) + q_i(x(\tau_i - 0), y(\tau_i - 0)).
\end{align*}
\]  

(1)

Sufficient conditions under which there is Lipschitzian map with respect to the second variable \( u: \mathbb{R} \times X \to Y \) are obtained. Using this result we reduce the investigation of stability of trivial solution to simpler equation

\[
\begin{align*}
\frac{dx}{dt} &= A(t)x + f(t, x, u(t, x)), \\
\Delta x\big|_{t = \tau_i} &= C_i x(\tau_i - 0) + p_i(x(\tau_i - 0), u(\tau_i - 0), y(\tau_i - 0)).
\end{align*}
\]  

(2)

---

**On the solvability of the Cauchy problem for linear functional differential equations**

András Rontó, DrSc

Institute of Mathematics, Academy of Sciences of the Czech Republic

Abstract. We discuss solvability conditions of the initial value problem for systems of linear functional differential equations of the general form. Several corollaries of the general theorems are given.

**How noise and spatial distribution produce the stabilization of prey-predator populations**

Marcelo Schiffer

Dept. Physics, University Center of Ariel, Israel

Two interacting species in a prey-predator behavior are placed at two spatial patches. Stability issues are discusses. It is shown that the effects of migration and noise alone cannot account for the stabilization. Then we show that the ingredients required for the stability observed in Nature are: spatial distribution; migration among the sites; dependence of the period of variance of the population upon the size of the population (amplitude) and noise. A simple model of diffusively coupled oscillators with amplitude dependent in the presence of noise is explicitly displayed and shown how desynchronization, a crucial condition for stabilization, is produced under these hypothesis.

Amplitude of the oscillations are shown to be (almost) noise independent.
New Results for Solutions of Volterra Difference Equations with Periodic Coefficients

Ewa Schmeidel
Poznań University of Technology, Poland

In many real-life situations, the present state and the manner in which it changes are both dependent on the past. In population dynamics, the birth-rate is clearly a function of the state of the population at some previous date, as well as its current state, because of the significance of the gestation period. Volterra difference equations, whose solution is defined by the whole previous history, are widely used for modeling processes in many fields.

Sufficient conditions for the existence of weighted asymptotically periodic solutions of Volterra difference equations with periodic coefficients are presented.

Homogenization of the diffusion equation with nonlinear flux condition on the interior boundary of a perforated domain - the influence of the scaling on the nonlinearity in the effective sink-source term

T.A. Shaposhnikova
Moscow State University, Moscow, Russia

We study the asymptotic behavior of solutions $u_\varepsilon$ of the initial boundary value problem for that parabolic equations in perforated domains $\Omega_\varepsilon \subset \mathbb{R}^n$, $n \geq 3$, with nonlinear third type boundary condition $\partial_{\nu} u_\varepsilon + \varepsilon^{-\alpha} \sigma(x, u_\varepsilon) = \varepsilon^{-\alpha} g(x)$ on the boundary of the cavities. It is supposed that the perforations are balls of radius $\varepsilon^\alpha$, $\alpha = n/(n-2)$, periodically distributed with period $\varepsilon$.

As $\varepsilon \to 0$, the microscopic solutions can be approximated by the solution of an effective equation on the domain $\Omega$. The effective equation contains a new sink/source term representing the macroscopic contribution of the processes on the boundary of the microscopic cavities.
INTERVAL ABSOLUTE STABILITY OF SPECIAL TYPE FUNCTIONAL-DIFFERENTIAL SYSTEMS WITH AFTEREFFECT
ANDRIY SHATYRKO *

Abstract. Investigations on absolute stability of nonlinear regulator systems (stability as a whole of the trivial solution for the set class of nonlinear characteristics) take the sources from the middle of the last century. Utilized of Lyapunov’s method with use as Lyapunov’s functions, and the functional of Lyapunov-Krasovskiy, algebraic criteria of absolute stability of the nonlinear regulator systems described in terms of the differential-functional equations with time-delay argument and neutral type are received. Analogous results are developed on a case of systems with inexact defined parameters of linear part of system (interval stability). Estimates of the exponential decay of solution and coefficients interval uncertainties have been calculated [1,2].

REFERENCES


Key Words. Nonlinear regulator system, time delay argument, Lyapunov’s method

AMS(MOS) subject classification. 34C10, 34K20, 34K05, 34K35, 93D10


Efraim Shmerling

Department of Computer Science and Mathematics
Ariel University Center of Samaria, Israel

Abstract

Asymptotic stability of the zero solution for linear systems of differential Equations with Markovian and Semi-Markovian switching is examined. Systems of Matrix equations are derived for which the existence of a positive definite solution implies the asymptotic stability of the considered systems with switching. Finally, illustrative examples are presented.
Existence Conditions for Solutions of Weakly Perturbed Linear Impulsive Systems Bounded on the Entire Real Axis

Martina Langerová, Jaroslava Škoríková
martina.langerova@fpv.uniza.sk, jaroslava.skorikova@fpv.uniza.sk

Abstract. We consider the linear weakly perturbed impulsive systems in the form
\[ \dot{x} = A(t)x + \varepsilon A_1(t)x + f(t), \quad A(.), A_1(.) \in BC(\mathbb{R}), \]

\[ \Delta x \mid_{t = \tau_i} = \gamma_i + \varepsilon A_{1i}(\tau_i - 0), \quad t, \tau_i \in \mathbb{R}, i \in \mathbb{Z}, \quad \gamma_i \in \mathbb{R}^n. \]

We assume that the unperturbed system has not solutions bounded on the entire real axis for arbitrary nonhomogeneities. By using the Vishik-Lyusternik method we establish conditions for the existence of solutions of these systems bounded on the entire real axis in the form of Laurent series in powers of small parameter \( \varepsilon \) with finitely many terms with negative powers of \( \varepsilon \).

Keywords: impulsive system, \( \varepsilon \)-dichotomy, Laurent series, bounded solutions, Vishik-Lyusternik method.

AMS Subject Classification: 34B05, 34B37.

Acknowledgment. The research was supported by the Grant 1/0771/08 of the Grant Agency of Slovak Republic (VEGA) and project APVV-0700-07 of Slovak Research and Development.

On the Darboux problem for linear functional-differential equations of hyperbolic type

Jiří Šremr

Czech Academy of Science, Brno, Czech Republic

On the rectangle \( \mathcal{D} = [a, b] \times [c, d] \), we will consider the functional-differential equation
\[ u''_{[12]}(t, x) = \ell_0(u)(t, x) + \ell_1(u'_{[1]})(t, x) + \ell_2(u'_{[2]})(t, x) + q(t, x), \]

where \( \ell_0: X_0 \to L(\mathcal{D}), \ell_1: X_1 \to L(\mathcal{D}), \ell_2: X_2 \to L(\mathcal{D}) \) are linear bounded operators, \( q \in L(\mathcal{D}) \), and \( X_0, X_1, X_2 \) are suitable function spaces. We shall discuss the question how to choose spaces \( X_0, X_1, X_2 \) to be able to use the concept of Carathéodory solutions for the study of equation (1). We shall remind that every solution to the hyperbolic equation
\[ u''_{[12]} = p_0(t, x)u + p_1(t, x)u'_{[1]} + p_2(t, x)u'_{[2]} + q(t, x) \]

with integrable coefficients admits a certain integral representation containing the so-called Riemann functions. Moreover, we shall present several solvability conditions for the Darboux problem for two-term equation (1) (i.e., if \( \ell_1 = 0 \) and \( \ell_2 = 0 \)).

\footnote{Symbols \( u'_{[1]} \) (resp. \( u'_{[2]} \)) and \( u''_{[12]} \) denote the first-order partial derivative with respect to the first (resp. second) variable and the second-order mixed partial derivative with respect to the first and then to the second variable.}
Singular fractional boundary value problems
Svatoslav Staněk
Palacky University, Olomouc, Czech Republic

We investigate the existence of positive solutions of the singular fractional boundary value problem
\[
cD^\alpha u(t) + f(t, u(t), u'(t), cD^\mu u(t)) = 0
\]
\[
u'(0) = 0, \quad u(1) = 0,
\]
where \(cD\) is the Caputo fractional derivative, \(\alpha \in (1, 2)\) and \(\mu \in (0, 1)\). Here \(f\) is a \(L^q\)-Carathéodory function on \([0, 1] \times D, D \subset \mathbb{R}^3, q > \frac{1}{\alpha - 1}\), and \(f(t, x, y, z)\) may be singular at the value 0 of its space variables \(x, y, z\). The results are proved by regularization and sequential techniques. The solvability of auxiliary regular fractional problems is proved by a fixed point theorem of cone compression type due to Krasnosel’skii.
The theory of Shtyrm for the solutions of linear
the second order stochastics equations
O. Stanzhytskyi and I. Novak
Taras Shevchenko National University of Kyiv
e-mail: stom@mail.univ.kiev.ua,kavonk@ukr.net

We consider the following stochastics equations
\[
\ddot{x} + (p(t) + q(t)\dot{W}(t))x = 0,
\]
where \(x \in \mathbb{R}^n\), \(t \geq 0\), \(p(t), q(t)\)–are continuous functions, \(W(t)\)–is a standard Wiener process. We understand the equation (1) as the following stochastics system
\[
\begin{align*}
dx_1 &= x_2dt, \\
dx_2 &= -p(t)x_1dt - q(t)x_1dW(t).
\end{align*}
\]
We introduce the notion a zero for these equations and we study the oscillation their solutions.

**Theorem 1.** Let the following condition are valid:
1) \(p(t) \leq 0\);
2) \(\int_0^\infty q^2(t)dt < \infty\)

Then all the solutions of the equation (1) are not oscillation on the positive half-line.

Further we consider the special stochastics equations
\[
\ddot{x} + (a^2 + q(t)\dot{W}(t))x = 0,
\]
where \(q(t)\) satisfy to the condition 2) of theorem 1.

**Theorem 2.** All the the solutions of the equation (2) are oscillation on the positive half-line.
Convergence of the Solutions of a Discrete Equation with Two Delays

Josef Diblík, Miroslava Ružičková, Zuzana Šutá
josef.diblik@fpv.uniza.sk, miroslava.ruwickova@fpv.uniza.sk,
zuzana.suta@fpv.uniza.sk

Abstract. We consider a discrete equation
\[ \Delta y(n) = \beta(n)[y(n - j) - y(n - k)] \]  
with two integer delays \( k \) and \( j \), \( k > j > 0 \). We assume \( \beta: \mathbb{Z}_{n_0}^{\infty} - k \to (0, \infty) \) is a discrete function where \( \mathbb{Z}_{n_0}^{\infty} = \{n_0, n_0 + 1, \ldots\} \). A solution \( y = y(n) \) of (1) is convergent if it has a finite limit \( \lim_{n\to\infty} y(n) \). Criteria for convergence of all solutions of (1) are presented.

AMS Subject Classification: 34K25.

Acknowledgment. The investigation was supported by the project APVV-0700-07 of Slovak Research and Development Agency and by Grant no. 1/0090/09 of the Grant Agency of Slovak Republic (VEGA).

Global attractivity in single species models
Viktor Tkachenko
Institute of Mathematics, National Academy of Sciences,
Kiev, Ukraine

Key Words: 34K20, 39A30
Subject Classification: functional differential equation, difference equation, global attractivity

We study global attractivity of the zero solution for the nonlinear functional differential equation which arises in many contexts in mathematical biology
\[ x'(t) = -\delta x(t) + f(t, x_t), \quad x_t(\theta) = x(t + \theta), \theta \in [-h, 0], \delta > 0, \]
where functional \( f: \mathbb{R} \times C \to \mathbb{R}, C = C[-h, 0] \), satisfies Carathéodory condition and is estimated by rational or exponential function.
We consider also corresponding retarded differential equations with piece-wise constant argument and higher order difference equations.
On positive periodic solutions of singular nonlinear boundary value problems

Milan Tvrdý
Institute of Mathematics, Academy of Sciences of the Czech Republic, Žitná 25, CZ 115 67 Praha 1, Czech Republic, e-mail: tvrdy@math.cas.cz

2000 Mathematics Subject Classification: 34B16, 34C25, 34B15, 34B18.

Key words: Singular problem, periodic problem, p-Laplacian, repulsive singularity, antimaximum principle, quasilinear equation.

The contribution deals with the existence of solutions to the problem

\[(\phi_p(u'))' + a(t) \phi_p(u) = f(t, u), \quad u(0) = u(T), \quad u'(0) = u'(T), \quad (P)\]

where \(0 < T < \infty, 1 < p < \infty, \phi_p\) stands for the \(p\)-Laplacian, \(\phi_p(y) = |y|^{p-2} y\) for \(y \in \mathbb{R}\), \(a \in L_{\alpha}[0, T]\) for some \(\alpha, 1 \leq \alpha \leq \infty\), and \(f : [0, T] \times (0, \infty) \rightarrow \mathbb{R}\) is regular in \([0, T] \times (0, \infty)\), but can have singularity for \(x = 0\). A crucial moment is whether the corresponding quasilinear problem

\[(\phi_p(u'))' + a(t) \phi_p(u) = h(t), \quad u(0) = u(T), \quad u'(0) = u'(T), \quad (Q)\]

satisfies the anti-maximum principle, i.e. whether for each \(h \in L^1[0, T]\) such that \(h \geq 0\) a.e. on \([0, T]\), any solution of the problem \((Q)\) is nonnegative on \([0, T]\).

We will present new conditions ensuring that problem \((Q)\) fulfills the antimaximum principle and therefore a substantial condition ensuring the existence of a positive solution to \((P)\) is satisfied. Our main goal is to include the case that \(a\) can change its sign on the interval \([0, T]\). In a rather classical case that \(\alpha = \infty\) and \(a \geq 0\) a.e. on \([0, T]\) our condition reduces to a well-known condition \(\|a\|_{\infty} \leq \lambda_p^D\), where \(\lambda_p^D\) stands for the first eigenvalue of the related Dirichlet problem.
Controllability of Evolution Equations 
and exponentials families

Benzion Shklyar
Holon Institute of Thechnology, Israel
shk_b@hit.ac.il

Let $X, U$ be complex Hilbert spaces, where $U$ is finite-dimensional with dimension $r \geq 1$, and let $A$ be infinitesimal generator of strongly continuous $C_0$-semigroups $S(t)$ in $X$. Consider the abstract evolution control equation

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \quad 0 \leq t < +\infty, \quad (1)$$

where $x(t), x_0 \in X, u(t) \in U$ is a control function, $B : U \to X$ is a linear possibly unbounded operator, $W \subset X \subset V$ are Hilbert spaces with continuous dense injections, $W = D(A)$ equipped with graphic norm, $V = W^*$, the operator $B$ is a bounded operator from $U$ to $V$ (see more details in [1], [2]).

The exact null-controllability problem can be formulated roughly as follows.

Given predefined time $t_1$ and initial state $x_0$, the goal is to find out whether there exists an admissible control $u(t)$ driving $x_0$ to the zero final state, provided that a control will be turned off after predefined time $t_2$, $t_2 \leq t_1$.

Necessary and sufficient conditions of exact null-controllability for linear evolution control equations with unbounded input operator are are obtained by transformation of exact null-controllability problem to linear infinite moment problem, which is defined as follows.

Given sequences $\{c_n, n = 1, 2, \ldots, \}$ and $\{x_n \in X, n = 1, 2, \ldots, \}$ find an element $g \in X$ such that

$$c_n = (x_n, g), \quad n = 1, 2, \ldots, \quad (2)$$

where $(x, y), x, y \in X$ is the inner product of $X$.

In this talk we present the null-controllability of control evolution equations for the case when the sequence $\{x_n, n = 1, 2, \ldots, \}$ doesn’t form a Riesz basic in its closed linear span. Applications to linear functional differential neutral control systems are considered.

References
