
5th Israeli Czech Workshop on Functional Differential Equations

ABSTRACTS
Optimal Control with Bilinear Inequality Constraints

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Semi-active systems provide an attractive alternative to active and hybrid control systems for structural vibration reduction. For many semi-active devices, physical considerations constrain the actual damping force such that it can only resist the structural motion in the damper anchors. In order to derive an appropriate optimal control law, a dynamic optimization subjected to inequality constraints is required. In this study, Karush-Kuhn-Tucker conditions are used to find a candidate optimum for two cases - one control signal and two control signals. An algorithm is proposed for numerical implementation of the control signals as a piecewise linear feedback.
A Numerical Method to Solve Maxwell’s Equations in Singular Domains with Arbitrary Data

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We propose a new numerical method to solve the Maxwell equations in an axisymmetric singular 3D domain $\Omega$, generated by the rotation of a singular polygon $\omega$ around one of its sides. This domain is singular in the sense that it contains reentrant corner or edges. Due to the symmetry of the domain, we first consider the equations written in a $(r, \theta, z)$ geometry. However, the data are arbitrary, namely do not necessarily satisfy $\partial/\partial \theta = 0$. Hence, the problem cannot be reduced to a two-dimensional one.

Nevertheless, one can use a Fourier transform in $\theta$ to reduce the 3D Maxwell equations to a series of 2D Maxwell equations, depending on the Fourier variable $k$. Hence, we will compute the solution to the 3D Maxwell equations by solving a certain number of 2D problems, depending on $k$. Let us denote by $(E_k, B_k)$ the electromagnetic field solution for each mode $k$. Following [1] and [2], one can prove that $(E_k, B_k)$ are singular only for $k = \pm 2, \pm 1, 0$. In that case, they can be decomposed into a regular and a singular part. One gets, for instance for $B_k$,

$$B_k = B_k^R + B_k^S,$$

where the regular part $B_k^R$ belongs to a space of regularity $H^1$, in which one can compute a numerical approximation by a finite element method. The difficulty comes from the singular part $B_k^S$ that belongs to a finite-dimensional subspace, the dimension of which being related to the number of reentrant corners and edges of the 2D singular polygon $\omega$. We will propose a new
approach based on a decomposition of the computational domain into sub-domains, and will derive an ad hoc variational formulation, in which the interface conditions are imposed with a method deduced from a Nitsche approach.

In this talk, we will consider as an illustration the case $k = 0$. We will show how to compute $w^S_0$, the singular part of the magnetic field solution $B^S_0$. We will then derive the time-dependent variational formulation to compute $B^R_0$ and then reconstruct the entire solution $B_0$. A similar approach can be used to compute $E_0$. Examples to illustrate our method will be shown.

References


A system of Four Difference Equations for Exploring the Dynamics of Dengue Spread, and its Control (Preliminary Studies)

T. Awerbuch-Friedlander, R. Levins
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We are expanding a previous system of three difference equations (Awerbuch-Friedlander T., Levins R. and Predescu M. Far East Journal of Applied Mathematics 37, 2: 215-228, 2009) to include the proportion of infected people that prompt the awareness for intervening:

Awareness (A) is prompted by the proportion of sick people (P). Control of Mosquitoes (M) is carried out directly by spraying, or by community intervention through the habitats (H).

\[ \begin{align*}
P_{n+1} &= a^* P_n + [1 - \exp(-i^*M_n)]*(1- P_n) \\
M_{n+1} &= l^* M_n*\exp(-g^*A_n) + b^* H_n*(1- \exp(-s^*M_n)) \\
H_{n+1} &= c^* H_n/(1+p^*A_n) + d/(1+q^*A_n) \\
A_{n+1} &= r^* A_n + f^*P_n
\end{align*} \]

Preliminary results show that \((0, 0, 1/(1-c), 0)\) is a degenerate equilibrium point, saying that habitats, the waters that support mosquitoes are always there, when the other variables are zero;

Simulations show that not all the variables in the system exhibit the same dynamics. With simulations we investigated the role of memory, the parameter \(r\), in community awareness. When the memory parameter is large, the proportion of infected people decreases and stabilizes at zero. Below a critical point we observe periodic oscillations, where the peak of awareness lags one week behind the peak of the proportion of infected people.

There is also a positive equilibrium for \((P, M, H \text{ and } A)\). However its global asymptotical stability is an open mathematical problem that would be of interest to investigate.
New Global Exponential Stability Criteria for
Nonlinear Delay Differential Systems with
Applications to BAM Neural Networks

L. Berezansky

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We consider a nonlinear non-autonomous system with time-varying delays
\[ \dot{x}_i(t) = -a_i(t)x_i(h_i(t)) + \sum_{j=1}^{m} F_{ij}(t, x_j(g_{ij}(t))), \quad i = 1, \ldots, m \]
which has a large number of applications in the theory of artificial neural networks. Via the $M$-matrix method, easily verifiable sufficient stability conditions for the nonlinear system and its linear version are obtained. Application of the main theorem requires just to check whether a matrix, which is explicitly constructed using the system’s parameters, is an $M$-matrix. Comparison with the tests obtained by K. Gopalsamy (2007) and B. Liu (2013) for BAM neural networks illustrates novelty of the stability theorems.
Dynamical Economic Models: Existence and Stability of Equilibria

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In this report the most important dynamical economic models at the micro- and macro-level of analysis are considered. First, we consider the dynamical system of elementary economic exchange and formulate conditions of existence and stability of its equilibria. Second, a dynamical consensus model of social choice is considered. Unlike well known static models of social choice (Arrow, 1951, 1963, Sen, 1970), in this report I aim at construction of an alternative model of social choice based on the value-powered exchange of economic goods. I demonstrate below that under some natural hypotheses about individual demand and supply functions of goods, the social consensus is possible, i.e. there exist stable stationary points in multivariate systems of social exchange of economic goods. These stable stationary points are interpreted as the social consensus points in dialogic (or poly-logic) processes of social choice.

At the macro-level of economic analysis we consider two well known dynamical macroeconomic models: Tobin’s q (1969) model of investment market, and the Ramsey-Cass-Koupmans (1964) model of goods market. Equilibrium points of these systems are only saddle-path stable.

In practice, however, it is extremely difficult to reach these stationary points: any deviation from initial conditions which lie on the saddle-path, leads to disaggregation of these systems. Therefore we use the idea of dynamical stabilization by means of introduction of a special control function. In general, all considered economic systems can be written in the form of a nonlinear system of ordinary differential equations

\[ \dot{X}(t) = J(X(t)) - u(t), \]

where \( X(t) \) is a vector of state variables, \( Y \) is a vector of equilibrium values of state variables.
Then we define the control as follows

\[ u(t) = \int_0^t l(k, s) [X(s) - Y] \, ds, \]

where

\[ l(t, s) = \beta e^{-\alpha(t-s)}. \]

Then using the methodology of stability analysis for systems of ordinary differential equations (see [1]), we can demonstrate that equilibrium points of a modified system with the introduced control function is stable for any initial conditions from a certain neighborhood of an equilibrium point.

References
Modeling and Simulation of Urinary Bladder Carcinoma

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Urinary bladder carcinoma also known as Bladder Cancer (BC) is the seventh most common cancer worldwide. According to existing statistics, 80% of BC patients had occupational exposure to chemical carcinogens (rubber, dye, textile, or plant industry) or/and were smoking regularly during long periods of time. The carcinogens from the bladder lumen affect umbrella cells of the urothelium (epithelial tissue surrounding bladder) and then subsequently penetrate to the deeper layers of the tissue (intermediate and basal cells). It is a years-long process until the carcinogenic substance will accumulate in the tissue in the quantity sufficient to trigger DNA mutations leading to the tumor development.

In this talk, I propose a model of BC progression that includes the crucial processes involved in tumor growth. My collaborator (Dr. Kashdan Eugene from University College Dublin) and I simulated oxygen diffusion, carcinogen penetration and angiogenesis within the framework of the urothelial cell dynamics. The cell living cycle is modeled using discrete technique of Cellular Automata, while the continuous processes of carcinogen penetration and oxygen diffusion are described by the nonlinear diffusion-absorption equations. Our model yields a theoretical insight into all stages of BC development and growth with especial accent on two most common types of urinary bladder carcinoma: bladder polyps and carcinoma in situ. Our numerical simulations are in a good qualitative agreement with in vivo results reported in the corresponding medical literature.
Polynomial Quasisolutions Method for Some Linear Differential Difference Equations of Mixed Type

V. Cherepennikov

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In this talk we consider a scalar linear differential difference equation (LDDE) of mixed type

\[ \dot{x}(t) = (a_0 + a_1 t)x(t) + (b_0 + b_1 t)x(t-1) + (d_0 + d_1 t)x(t+1) + \bar{f}(t), \quad t \in \mathbb{R}, \]

where \( \bar{f}(t) = \sum_{n=0}^{F} \bar{f}_n t^n \). This equation is investigated with the use of the method of polynomial quasisolutions based on the representation of an unknown function in the form of polynomial \( x(t) = \sum_{n=0}^{N} x_n t^n \). As a result of substitution of this function into equation (1), there appears a residual \( \Delta(t) = O(t^N) \), for which an exact analytical representation has been obtained. In turn, this allows one to find the unknown coefficients \( x_n \) and consequently the polynomial quasisolution \( x(t) \).

Several examples are considered.
Probabilistic Methods for a Class of Equations

Rescaling

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The question about the existence and characterization of bounded solutions to linear functional-differential equations with both advanced and delayed arguments was posed in early 1970s by T. Kato in connection with the analysis of the pantograph equation, \( y'(x) = ay(qx) + by(x) \). In our talk, we answer this question for the balanced generalized pantograph equation of the form

\[
y'(x) + y(x) = \sum_i p_i y(\alpha_i x),
\]

under the balance condition \( \sum_i p_i = 1 \) \( (p_i \geq 0) \). Namely, setting \( K := \sum_i p_i \ln \alpha_i \), we prove that if \( K \leq 0 \) then the equation does not have nontrivial (i.e., nonconstant) bounded solutions, while if \( K > 0 \) then such a solution exists. The result in the critical case, \( K = 0 \), settles a long-standing problem. The proofs are based on the link with the theory of Markov processes and employ martingale technique. Same approach may be applied also for other types of equations with rescaling (i.e. functional, integral and integro-differential ones).

The talk is based on joint work with Leonid Bogachev (Leeds, UK), Stanislav Molchanov (North Carolina at Charlotte, USA) and John Ockendon (Oxford, UK).
Positivity and Stability of Solutions to Second Order Delay Differential Equations

A. Domoshnitsky

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Delays, arising in nonoscillatory and stable ordinary differential equations, can induce oscillation and instability of their solutions. That is why the traditional direction in the study of nonoscillation and stability of delay equations is to establish a smallness of delay, allowing delay differential equations to preserve these convenient properties of ordinary differential equations with the same coefficients. In this paper, we find cases in which delays, arising in oscillatory and asymptotically unstable ordinary differential equations, induce nonoscillation and stability of delay equations. We demonstrate that, although the ordinary differential equation \( x'' + c(t)x(t) = 0 \) can be oscillating and asymptotically unstable, the delay equation \( x''(t) + a(t)x(t - h(t)) - b(t)x(t - g(t)) = 0 \), where \( c(t) = a(t) - b(t) \), can be nonoscillating and exponentially stable. Results on nonoscillation and exponential stability of delay differential equations are obtained. On the basis of these results on nonoscillation and stability, the new possibilities of noninvasive (non-evasive) control, which allow us to stabilize a motion of single mass point, are proposed. Stabilization of this sort, according to common belief requires damping term in the second order differential equation. Results obtained in this paper refutes this delusion.
Maximum Principles for Functional Differential Equations with Nonlocal Boundary Conditions

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The objective of this talk is to present an approach to studying the maximum principles for functional differential equations. On this basis assertions about existence, uniqueness and sign properties of solutions to nonlocal boundary value problems, problems with nonseparating boundary conditions and nonlinear boundary conditions could be obtained. The idea of our approach is to construct a corresponding linear "model" problem of the order m possessing properties, which we want to find in the given problem of the order n. Substituting a solution’s representation of the "model" problem in the given one, we reduce obtaining maximum principles to analysis of positivity of solutions to a corresponding operator equation (when $n = m$) or to boundary value problems, which are more "convenient" for us (in the cases $m < n$ or $m > n$). For this analysis corresponding "nonoscillation" methods could be used. The idea of a substitution can be also used for hyperbolic partial differential equations. Although the given equation is partial differential, both equations (a "model" one and the differential equation after substitution) could be with ordinary derivatives only. Thus the Green’s operator of corresponding partial functional differential boundary value problem could be presented as a product of the Green’s operators of several problems with functional differential equations with ordinary derivatives. This allows us to use a technique developed for equations with ordinary derivatives for the study of boundary value problems with partial functional differential equations.
Lyapunov-Based Methods for Stability and Control of Time-Delay Systems

E. Fridman

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Time-delay naturally appears in many control systems, and it is frequently a source of instability. However, for some systems, the presence of delay can have a stabilizing effect. In this talk, two main Lyapunov-based approaches to stability of time-delay systems will be presented: Krasovskii method of Lyapunov functionals (1956) and Razumikhin method of Lyapunov functions (1956). A special attention will be put to the descriptor approach (Fridman, 2001), which allowed to solve new problems for time-delay systems and to improve the existing results via Krasovskii method.

This approach appeared to be useful for robust control of systems without delay both, in continuous and in discrete time. It also allowed to develop an input delay approach to robust sampled-data control and to networked control systems. Finally, some recent Lyapunov functionals for systems with a sawtooth delay (corresponding to sampled-data control) will be presented.
In NMR (nuclear magnetic resonance) one studies several types of exchange processes. In some cases of multiple-site chemical exchange the process is cyclic, with a characteristic effect on the NMR spectrum. If one generalizes the process by assuming non-negligible time for the exchange jumps, the system evolves by a set of delay differential equations. These equations are solved using the complex Lambert function. Some explicit results are given for the 3-site case, investigating the conditions for pure decay and the dominant oscillating terms. A comparison is made with the 2-site case.
In this talk the following results are presented:

1) The conditions of neutrality in linear approximation,

2) The relation between the structure of a Poincare normal forms and linear and nonlinear forces in damped oscillatory systems,

3) The stability and instability of the stationary points of the nonlinear oscillatory systems (in sense of Lyapunov or Birkhoff)

4) The bifurcations of the steady resonance modes. Example: gyroscopic wagon under the action external almost periodical forces.
On Stability of Some Oscillating System of Integro-Differential Equations

Y. Goltser

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In many works about stability the cases for which the integral terms are in some sense negligible and do not affect stability of the system are singled out. Here we consider a class of IDEs for which the nonlinear integral terms affect the asymptotic properties of stability or instability of IDEs. In this approach we essentially use, along with the reduction method, the method of normal forms, which proved its efficiency in the solution of local nonlinear problems of the qualitative theory of ODEs. The method of reduction of nonlinear IDEs can be essentially used also for the cases when the IDE system is affected by parametric disturbances. In this context an important situation is when we are close to the critical case, it is common in problems of bifurcation type. For example, such problems are arising in stationary regimes of resonance and non-resonance types, stability of IDEs under passage through the internal resonance, etc. The introduction of the notion of the internal resonance becomes apparent only after the reduction of the IDE system to ODEs and consideration of the spectrum of the system. To study stability of the zero solution are construct the quasi-normal form (QNF) for the countable ODE system.
Existence of Global Solutions to the Linear Functional Differential Equations on the Real Half-Line

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Consider the equation
\[ u'(t) = \ell_0(u(t)) - \ell_1(u(t)) + q(t) \quad \text{for a. e. } t \in \mathbb{R}_+ \]
subject to the condition
\[ u(0) = c, \]
where \( \ell_i : C_{\text{loc}}(\mathbb{R}_+; \mathbb{R})_{\text{loc}}(\mathbb{R}_+; \mathbb{R}) \) \( (i = 0, 1) \) are linear positive continuous operators, \( q_{\text{loc}}(\mathbb{R}_+; \mathbb{R}) \), and \( c \in \mathbb{R} \). The efficient conditions guaranteeing the existence of a global solution to the equation considered are established. The existence of a solution which are non-negative and/or non-decreasing on the whole real half-line is discussed, as well. The results will be reformulated for the equation with deviating arguments of the form
\[ u'(t) = p_0(t)u(\mu_0(t)) - p_1(t)u(\mu_1(t)) + q(t) \quad \text{for a. e. } t \in \mathbb{R}_+ \]
where \( p_{\text{loc}}(\mathbb{R}_+; \mathbb{R}_+) \) and \( \mu_i : \mathbb{R}_+ \to \mathbb{R}_+ \) are locally essentially bounded functions \( (i = 0, 1) \).
Existence and Properties of Semi-Bounded Global Solutions to the Functional Differential Equation with Volterra’s Type Operators on the Real Line

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M. Aguerrea  
UCM, Chile

Consider the equation

\[ u'(t) = \ell_0(u)(t) - \ell_1(u)(t) + f(u)(t) \quad \text{for a. e. } t \in \mathbb{R} \]

where \( \ell_i : C_{\text{loc}}(\mathbb{R}; \mathbb{R})_{\text{loc}}(\mathbb{R}; \mathbb{R}) \) \( (i = 0, 1) \) are linear positive continuous operators and \( f : C_{\text{loc}}(\mathbb{R}; \mathbb{R})_{\text{loc}}(\mathbb{R}; \mathbb{R}) \) is a continuous operator satisfying the local Carathéodory conditions. The efficient conditions guaranteeing the existence of a global solution, which is bounded and non-negative in the neighbourhood of \(-\infty\), to the equation considered are established provided \( \ell_0, \ell_1, \) and \( f \) are Volterra’s type operators. The existence of a solution which is positive on the whole real line is discussed, as well. Furthermore, the asymptotic properties of such solutions are studied in the neighbourhood of \(-\infty\). The results are applied to certain models appearing in natural sciences.
Comparison Theorems for Second Order Linear
Differential Equations

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For singular linear differential equations comparison theorems are given and some two-point singular boundary value problems for second order linear differential equations are investigated.

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Differential Equations in Algebras

Y. Krasnov

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The main purpose of the talk is to show how the algebraic formalism can be applied with great success to a remarkably elegant description of the geometry of curves being solutions to homogeneous polynomial ODEs as well as to arouse interest to the algebraic language in PDEs.
Criteria for Invariance of Convex Bodies for Linear Parabolic Systems

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We consider systems of linear partial differential equations, which contain only second and first derivatives in the $x$ variables and which are uniformly parabolic in the sense of Petrovskii in the layer $\mathbb{R}^n \times [0, T]$. For such systems we obtain necessary and, separately, sufficient conditions for invariance of a convex body. These necessary and sufficient conditions coincide if the coefficients of the system do not depend on $t$. The above mentioned criterion is formulated as an algebraic condition describing a relation between the geometry of the invariant convex body and coefficients of the system. The criterion is concretized for certain classes of invariant convex sets: polyhedral angles, cylindrical and conical bodies.
On Existence of a Positive Solution of an Homogeneous Linear Functional Differential Equation

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Consider the problem

\begin{align*}
(-1)^n x^{(n)}(t) - \sum_{i=0}^{n-1} \int_0^\infty (-1)^i x^{(i)}(s)d_i(t,s) &= 0, \quad t \in R_+ = [0, \infty), \quad (1) \\
x(t) > 0, \quad (-1)^i x^{(i)}(t) &\geq 0 \quad (i = 1, \ldots, n - 1, \quad t \in R_+). \quad (2)
\end{align*}

Assume that for all \(i \in \{0, \ldots, n - 1\}, \ s \in R_+\) the function \(r_i(\cdot, s)\) is measurable on \(R_+\), for almost all \(t \in R_+\) the function \(r_i(t, \cdot)\) does not decrease on \(R_+\). A solution of the equation (1) is a function \(x \in D_{loc}(R_+)\) satisfying (1) for almost all \(t \in R_+\). Assume that \(r_i(t, 0) = 0\).

Let \(L_{loc}(R_+)\) be the set of functions \(z: R_+ \rightarrow R\) locally Lebesgue integrable on \(R_+\), \(D_{loc}(R_+)\) be the set of functions \(x: R_+ \rightarrow R\) locally on \(R_+\) absolutely continuous together with its derivatives up to and including order \(n - 1\).

**Theorem 0.1** (1). *Let the functions \(r_i(t, s), \ i = 0, \ldots, n - 1\) satisfy the inequalities*

\begin{align*}
\limsup_{s \rightarrow +\infty} s^{-i} r_i(\cdot, s) &\in L_{loc}(R_+), \quad t \in R_+, \ i = 0, \ldots, n - 1, \quad (3) \\
\int_0^\infty t^{n-1} r_0(t, \infty) &= +\infty. \quad (4)
\end{align*}
Suppose there exists a function $x \in D_\text{loc}(R_+)$ satisfying the inequalities (2) and one of the inequalities

$$(-1)^n x^{(n)}(t) - \sum_{i=0}^{n-1} \int_0^\infty (-1)^i x^{(i)}(s) d_x r_i(t, s) \geq 0, \quad t \in R_+, \quad (5)$$

or

$$x(t) \geq \int^\infty_t (s-t)^{n-1} \frac{1}{(n-1)!} \sum_{i=0}^{n-1} \int_0^\infty (-1)^i x^{(i)}(s) d_x r_i(s, \tau) ds, \quad t \in R_+. \quad (6)$$

Then there exists a solution of the problem (1),(2).
About Sign-Constancy of Green’s Functions for Impulsive Second Order Delay Equations

A. Domoshnitsky

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We consider a second order delay differential equation with impulses. In this paper we find necessary and sufficient conditions of positivity of Green’s functions for this impulsive equation coupled with one or two-point boundary conditions in the form of theorems about differential inequalities. By choosing the test function in these theorems, we obtain simple sufficient conditions.
Multiplicty Results for Higher Order Differential
BVPs and Integral Equations

F. Minhós

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In this talk it is presented two types of multiplicty results:
1. A parameter discussion for existence, non-existence and multiplic-
ity to a fourth order fully nonlinear problems, by lower and upper solutions
technique, a priori estimations and topological degree theory. The method
replaces the usual bilateral Nagumo condition by one-sided case. An appli-
cation to a continuous model of the human spine, used in aircraft ejections,
vehicle crash situations and some forms of scoliosis, will be presented;
2. Sufficient conditions for the existence of multiple solutions for a
wide class of systems of BVPs with a coupling boundary conditions. The
approach relies on a classical fixed point index theory, and as boundary
conditions are fairly general it covers a large number of situations.
Localization Results and Extremal Solutions for Higher Order Functional BVPs

F. Minhós

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In this talk it is presented two types of results:
1. Sufficient conditions for the existence of solutions for a nth order BVP with functional dependence, not only in the differential equation, but also on the boundary conditions. Applying lower and upper solutions method, some location sets for the solutions and its derivatives up to order (n-1) are established. Moreover, it is shown how the monotone properties of the nonlinearity and the boundary functions depend on $n,$ and on the relation between lower and upper solutions and their derivatives;
2. Existence of extremal solutions to a fourth order functional problem composed by a nonlinear equation, together with functional boundary conditions, both not necessarily continuous, satisfying some monotonicity assumptions. The arguments make use of lower and upper solutions technique, a version of Bolzano’s theorem and existence of extremal fixed points for a suitable mapping.
Remarks on Continuous Dependence of Solution of Abstract Generalized Differential Equations

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Generalized differential equations were introduced in 1957 by J.Kurzweil. In particular, it was a problem on continuous dependence that inspired him to extend the notion of classical ODE’s.

In this work, we discuss continuous dependence results for generalized differential equations with a particular interest in the linear case. More precisely, we investigate integral equations of the form

\[ x(t) = \tilde{x}_k + \int_a^t d[A_k] x + f_k(t) - f_k(a), \quad t \in [a, b], \quad k \in \mathbb{N}, \]

where \( A_k : [a, b] \rightarrow L(X) \) have bounded variations on \( [a, b] \), \( f_k : [a, b] \rightarrow X \) are regulated on \( [a, b] \) and \( \tilde{x}_k \in X \), with \( X \) being a Banach space.

Herein we pay special attention to recently published results found in [2], where we extend Theorem 4.2 from [1] to the non-homogeneous case. In addition, we provide an example showing that the obtained conditions are somehow optimal.

This is a joint work with M. Tvrdý.

References


Linear Measure Functional Differential Equations with Infinite Delay via Generalized Differential Equations

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The notion of measure functional differential equation was introduced in [1] and, since then, it has been investigated by many authors. In [3], equations of this type with infinite delay were studied and its relation with the theory of generalized ordinary differential equations, introduced by J. Kurzweil, was established. Based on that, our aim is to discuss continuous dependence results for linear functional equations of the form

$$y(t) = y(a) + \int_a^t \ell(y_s, s) \, dg(s) + \int_a^t p(s) \, dg(s), \quad t \in [a, b],$$

by the means of generalized linear differential equations.

This is a joint work with A. Slavík and it corresponds to the results found in [2].

References


Two-Point Boundary Value Problems For Strongly Singular Higher-Order Linear Differential Equations With Deviating Arguments

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Consider the differential equation with deviating arguments
\[ u^{(n)}(t) = \sum_{j=1}^{m} p_j(t)u^{(j-1)}(\tau_j(t)) + q(t) \quad \text{for} \quad a < t < b, \tag{1} \]
with the two-point boundary conditions
\[ u^{(i-1)}(a) = 0 \quad (i = 1, \ldots, m), \quad u^{(j-1)}(b) = 0 \quad (j = 1, \ldots, n-m). \tag{2} \]
Here \( n \geq 2, \ m \) is the integer part of \( n/2 \), \( -\infty < a < b < +\infty \), \( p_j, q \in L_{\text{loc}}[a, b] \) \( (j = 1, \ldots, m) \), and \( \tau_j : [a, b] \to [a, b] \) are measurable functions. By \( u^{(j-1)}(a) \) \( (u^{(j-1)}(b)) \) we denote the right (left) limit of the function.

We study problem (1), (2) in the case when the functions \( p_j \) and \( q \) have strong singularities at the points \( a \) and \( b \), i.e. when the conditions
\[
\int_{a}^{b} (s-a)^{n-1}(b-s)^{2m-1}(-1)^{n-m}p_1(s) \, ds < +\infty,
\]
\[
\int_{a}^{b} (s-a)^{n-j}(b-s)^{2m-j}|p_j(s)| \, ds < +\infty \quad (j = 2, \ldots, m),
\]
\[
\int_{a}^{b} (s-a)^{n-m-1/2}(b-s)^{m-1/2}|q(s)| \, ds < +\infty,
\]
are not fulfilled. In this case the Agarwal-Kiguradze type theorems are proved which guarantee Fredholm’s property for problems (1), (2), moreover, we establish in some sense optimal, sufficient conditions for the solvability of the mentioned problem.
About Positivity of the Cauchy Function for a Singular Functional Differential Equation

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The Cauchy problem

\[(Lx)(t) = \dot{x}(t) + \frac{k}{t} x \left(\frac{t}{\rho}\right) + (Tx)(t) = f(t), \quad t \in [0, b], \]  
\[x(0) = 0 \]  

(1)

(2)

with \( \rho > 1 \) and Volterra operator \( T \) is considered.

The theorem about differential inequality was formulated. The sufficient conditions of Cauchy function positivity were obtained.
About Solvability of the Cauchy Problem for a Quasilinear
Singular Functional Differential Equation

I. Plaksina

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The semi-homogeneous Cauchy problem

\[(\mathcal{L}x)(t) \equiv \ddot{x}(t) + \frac{m}{t^2}x\left(\frac{t}{\rho}\right) = f(t, x(t)), \quad t \in [0, b], \quad (1)\]

\[x(0) = 0, \quad \dot{x}(0) = 0 \quad (2)\]

for the independent variable singular equation with delay of the special type was considered. Solvability conditions were obtained by the properties of generalized Cesaro operator.
About Functional Differential Equation with Delay on the Real Axis

V. Plaksina

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Conditions of unique solvability of the boundary-value problem

\[ \ddot{x}(t) + kx(t - \omega) + (Tx)(t) = f(t), \quad t \in (-\infty, +\infty), \]

\[ \lim_{t \to -\infty} x(t) = 0 \]

were obtained.
Exponential Stabilization of Unstable Fix Point in an Electrochemical System by Feedback Control in Integral Form

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In this work we consider a model for aqueous electrochemical corrosion in the electrochemical cell. This model described by a system of three dimensionless differential nonlinear equations proposed in [1]. To stabilize the unstable fixed point the authors of [1] use an additional differential equation with derivative control of chemical reaction rate.

We use another way to construct the control function in an integral form:

\[ u(t) = \int_0^t k(t, s) X(s) \, ds, \]

in which all the history of the process \( X(t) \) is taken into account. Using the exponential kernel \( k(t, s) = e^{-\beta(t-s)} \), we reduce the study of integro-differential system of the order 3 to analysis of 4-th order system of ordinary differential equations.

Numerical solution of resulting system leads to the exponential stabilization of unstable fixed point with almost the same limit values of the electrochemical variables were obtaind in [1].

In this work, we demonstrate that the problem of controlling chaos, which is of great theoretical and practical importance, can be reduced to the stability analysis of corresponding integro-differential equations. We consider an unstable single-mode solid-state laser with periodically varied losses and stabilize it by the control function in integral form. In order to obtain stability results, we propose a special technique which based on the idea of reduction of integro-differential equations to system of ordinary differential equations.

Our approach is based on the generalized method of control function in integral form defined as 

\[ u(\tau) = \int_0^\tau e^{-\beta(\tau-s)} I(s) \, ds, \]

where \( I(s) \) is the dimensionless laser intensity. In order to obtain stability results, we reduce integro-differential equations to the system of ordinary differential equations with periodic coefficients. To find the control parameters obtained for non-autonomous system, we use the method of monodromy matrices. This method allows to solve the problem without special adjustment of the controlling parameters.
State-Dependent and Fixed-Time Impulsive BVPs

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We compare two types of impulsive BVPs - problems with impulses at fixed moments and problems with impulses which occur at moments depending on solutions of given equations. Similarities and basic differences between these two types are discussed. Then we investigate the boundary value problem

\[ z'(t) = f(t, z(t)), \quad \text{a.e. } t \in [a, b] \subset \mathbb{R}, \quad \ell(z) = c_0, \]  

with the state-dependent impulses

\[ z(t+) - z(t-) = J_i(t, z(t-)), \quad i = 1, \ldots, p, \]  

where the impulse instants \( t \in (a, b) \) are determined as solutions of the equations

\[ t = \gamma_i(z(t-)), \quad i = 1, \ldots, p. \]  

We assume that \( n, p \in \mathbb{N}, \ c_0 \in \mathbb{R}^n \), the vector function \( f \) satisfies the Carathéodory conditions on \([a, b] \times \mathbb{R}^n\), the impulse functions \( J_i, i = 1, \ldots, p \), are continuous on \([a, b] \times \mathbb{R}^n\), and the barrier functions \( \gamma_i, i = 1, \ldots, p \), are continuous on \( \mathbb{R}^n \). The operator \( \ell \) is an arbitrary linear and bounded operator on the space of left-continuous regulated (i.e. having finite one-sided limits at each point) on \([a, b]\) vector valued functions and is represented by the Kurzweil-Stieltjes integral. Provided the data functions \( f \) and \( J_i \) are bounded, transversality conditions which guarantee that this fixed point problem is solvable are presented. As a result it is possible to realize a construction of a solution of the above impulsive problem.
The Bäcklund transformations for the Camassa-Holm equation

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The Bäcklund transformation (BT) for the Camassa-Holm (CH) equation was found. The transformation was extended for a large class of equations which includes: Camassa-Holm, Hunter-Saxton and Korteweg-de Vries equations. With the help of BT we have constructed the nonlinear superposition principle for CH. The classification of one and two wave solutions was done. BT allows us to construct the infinite hierarchies of symmetries and conservation laws for CH.
Large Time Behavior of a Linear Delay Differential Equation with Asymptotically Small Coefficient

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We consider non-autonomous scalar linear functional differential equations with asymptotically small coefficient, and show that the large time behavior of solutions can be described in terms of a special solution of the associated formal adjoint equation and the initial data. In many cases this special solution can be constructed (or approximated), and then our results yield an explicit asymptotic representation of all solutions of the original equation.

Applications include for example the Dickman-de Bruijn equation.
This is a joint work with Mihály Pituk (Pannonia University, Veszprém, Hungary).
Constructive Study of Linear Functional Differential Equations with Distributed Delay

A. Rumyantsev

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ITPS Group, Perm, Russia

Report devoted to differential equations with distributed delay:

\[ \dot{x}(t) + \int_0^t x(s) d_s R(t, s) = f(t), \quad t \in [0, 1], \]

under the following assumptions: \( f : [0, 1] \to \mathbb{R}, \) \( f \in L[0, 1], \) \( L[0, 1] \) is the Banach space of summable functions \( y : [0, 1] \to \mathbb{R}, \) \( \|y\|_{L[0, 1]} = \int_0^1 |y(s)| ds, \)

\( x : [0, 1] \to \mathbb{R}, \) \( x \in D[0, 1], \) \( D[0, 1] \) is the Banach space of absolutely continuous functions \( x : [0, 1] \to \mathbb{R}, \) \( \|x\|_{D[0, 1]} = |x(0)| + \|\dot{x}\|_{L[0, 1]}; \) function

\( R : [0, 1] \times [0, 1] \to \mathbb{R} \) has the form

\[ R(t, s) = \sum_{i=1}^n u_i(t)v_i(s)\chi_i(t, s), \quad (t, s) \in [0, 1] \times [0, 1], \]

\( u_i, v_i \) are polynomials with rational coefficients, \( \chi_i \) is the characterized function of the set \( \{ (t, s) \in [0, 1] \times [0, 1] : 0 \leq s \leq h_i(t) \leq t \}, \) \( h_i : [0, 1] \to \mathbb{R} \) is the linear function with rational coefficients. A computer oriented method for studying the solvability of the boundary value problem

\[ \dot{x}(t) + \int_0^t x(s) d_s R(t, s) = f(t), \quad t \in [0, 1], \]

\[ \phi x(0) + \int_0^1 \psi(s) \dot{x}(s) ds = \alpha \]

where \( \alpha, \phi \) are rational numbers, \( \psi \) is the polynomial with rational coefficients, is proposed. Illustrative examples are given.
Constructive Approach to the Study of the Solvability of Linear Boundary Value Problems for Functional Differential Equations

A. Rumyantsev

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Part 1. Some facts from the theory of functional differential equations and general description of the constructive approach to the investigation of boundary value problems for such equations provides.

Part 2. The constructive approach to the investigation of the solvability of the linear boundary value problem for the second order ordinary differential equation described. The boundary value problem

\[ \ddot{x}(t) + p(t)x(t) = f(t), \quad t \in [0, 1], \]
\[ x(0) = x(1) = 0; \]
\[ p, f : [0, 1] \to R \in L[0, 1], \quad x : [0, 1] \to R \in W^2[0, 1], \]

is considered. Here \( L[0, 1] \) is the Banach space of summable functions \( y : [0, 1] \to R, \|y\|_{L[0,1]} = \int_0^1 |y(s)| \, ds \), \( W^2[0, 1] \) is the Banach space of functions \( x : [0, 1] \to R \) such that \( \ddot{x} \in L[0, 1], \|x\|_{W^2[0,1]} = \max \{ |x(0)|, |\dot{x}(0)| \} + \|\ddot{x}\|_{L[0,1]} \). Illustrative examples are given.

Part 3. The constructive approach to the investigation of the solvability of the linear boundary value problem for the second order differential equation with concentrated delay described. The boundary value problem

\[ \ddot{x}(t) + \sum_{i=1}^{n} p_i(t)x[h_i(t)] = f(t), \quad t \in [0, 1], \]
\[ x(\xi) = \varphi(\xi), \quad \xi \notin [0, 1], \]
\[ x(0) = x(1) = 0; \]
\[ p_i, f \in L[0, 1], \quad x \in W^2[0, 1], \]

is considered. Illustrative examples are given.
Let \( u_\varepsilon \) be the solution of the Poisson equation in a domain \( \Omega_\varepsilon \subset \mathbb{R}^3 \) perforated by thin tubes with a nonlinear Robin type boundary condition on the boundary of tubes (the flux here being \( \beta(\varepsilon)\sigma(x, u_\varepsilon) \)), and with a Dirichlet condition on the rest of the boundary of \( \Omega_\varepsilon \). \( \varepsilon \) is a small parameter that we shall make to go to zero; it denotes the period of a grid on a plane where the tubes have their bases; the size the transversal section of tubes is \( O(a_\varepsilon) \) with \( a_\varepsilon << \varepsilon \). A certain non-periodicity is allowed for the distribution of the thin tubes which have a fixed perimeter \( a \). The function \( \sigma \) involving the nonlinear process is a \( C^1(\Omega \times \mathbb{R}) \) function and the adsorption parameter \( \beta(\varepsilon) \) is an order function that can converge towards \( \infty \) as \( \varepsilon \to 0 \). Depending on the relations between the three parameters \( \varepsilon, a_\varepsilon \) and \( \beta(\varepsilon) \) the effective equations in volume are obtained. Among the multiple possible relations between these parameters, we provide critical relations which imply different averages of the process ranging from linear to nonlinear. After constructing a suitable extension of \( u_\varepsilon \) to \( \Omega \) we show the convergence as \( \varepsilon \to 0 \) towards that of the homogenized problem. All this allows us to derive convergence for the eigenvalues of the associated spectral problems in the case of \( \sigma \) is a linear function.
The goal of this talk is to obtain stability of uncertain systems and to estimate the difference between solutions of a real system with uncertain coefficients and/or delays and corresponding "model" system.

We develop the so-called Azbelev’s W-transform, which is a sort of the right regularization allowing researchers to reduce analysis of boundary value problems to study of systems of functional equations in the space of measurable essentially bounded functions. In corresponding cases estimates of norms of auxiliary linear operators (obtained as a result of W-transform) lead researchers to conclusions about existence, uniqueness, positivity and stability of solutions of given boundary value problems.

This method works efficiently in the case when a model used in W-transform is "close" to a given real system.

In this talk we choose, as the models systems for which we know estimates of the resolvent (Green’s) operators. We demonstrate that systems with positive Cauchy matrices present a class of convenient models.
System of Difference Equations for Defining the
Moments of Markov Order m Geometric Order k
Random Variables

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A system of difference equations for the probabilities of non-occurrence of a success run of length k in first n trials with success probability in each trial dependent on m previous trials is derived. Simple expressions for the cumulative distribution function and moments about the origin for Markov order m geometric order k random variables are obtained utilizing the system. Illustrative examples are given.
On Conjugacy Criteria for Linear Second-Order Differential Equations

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On the real line, we consider the linear second-order equation

\[ u'' + p(t)u = 0 \]

(1)

with a locally Lebesgue integrable function \( p: \mathbb{R} \to \mathbb{R} \). As usual, we say that equation (1) is conjugate on \( \mathbb{R} \) if it has a nontrivial solution with at least two zeros.

In 1978, Tipler proved (see [3]) that equation (1) is conjugate on \( \mathbb{R} \) provided

\[ \liminf_{t \to +\infty} \int_t^\infty p(s)ds > 0. \]

Later, this result was generalised in various ways. For instance, conjugacy of (1) can be described in terms of behaviour of the function

\[ t \mapsto \frac{1}{|t|} \int_0^t \left( \int_0^s p(\xi)d\xi \right) ds \]

(2)

in the neighbourhoods of \( +\infty \) and \( -\infty \) (see, e.g., [2]). In the paper [1], the authors considered an expression of type (2) in the more convenient symmetric form

\[ c(t) := \frac{1}{t} \int_0^t \left( \int_{-s}^s p(\xi)d\xi \right) ds \quad \text{for } t > 0 \]

and proved, among others, that equation (1) is conjugate on \( \mathbb{R} \) if

\[ \liminf_{t \to +\infty} c(t) \geq 0. \]
However, this result can be refined which allows one to continue in the investigation of conjugacy of (1) and to derive new conjugacy criteria under the natural additional assumption

\[-\infty < \lim_{t \to +\infty} c(t) < 0.\]

The aim of our talk is to present new “point-wise” conjugacy criteria formulated in terms of behaviour of the function $c$ on the entire interval $(0, +\infty)$ as well as new “integral” conjugacy criteria obtained by a certain series of transformations.

References


On the Hartman-Wintner Theorem for Half-Linear
Emden-Fowler Type Systems

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On the half-line \([0, +\infty[\), we consider the two-dimensional half-linear
Emden-Fowler type system

\[
\begin{align*}
    u' &= g(t)|v|^{1/\alpha} \text{sgn } v, \\
    v' &= -p(t)|u|^\alpha \text{sgn } u,
\end{align*}
\]

(1)

where \(g, p: [0, +\infty[ \to \mathbb{R}\) are locally integrable functions, \(\alpha > 0\), and \(g(t) \geq 0\)
for a.e. \(t \geq 0\). A frequently studied particular case of system (1) is the
half-linear second-order differential equations with the \(q\)-Laplacian

\[
(r(t)\Phi_q(u'))' + c(t)\Phi_q(u) = 0
\]

(2)

in which \(\Phi_q(x) := |x|^{q-1} \text{sgn } x, \ q > 1, \ r, c: [0, +\infty[ \to \mathbb{R}\) are continuous
functions, and \(r\) is positive.

The Hartman-Wintner theorem for equation (2) is well-known in the case,
where

\[
\int_0^{+\infty} r^{1/q}(s)ds = +\infty,
\]

(3)

which allows one to derive further oscillation and non-oscillation criteria of
Hille and Nahari type (see, e.g., [1, 3, 4] and references therein). As for the
case, where

\[
\int_0^{+\infty} r^{1/q}(s)ds < +\infty,
\]

(4)
as far as we know, the Hartman-Wintner theorem and some Hille and Nahari
type oscillation criteria are proved only under the additional assumption that
\(c(t) \geq 0\) for \(t \geq 0\) (see, e.g., survey given in [1]).
The aim of our talk is to present the Hartman-Wintner theorem for system (1), which generalises known results in the case, where \( \int_0^{+\infty} g(s)ds = +\infty \) (corresponding to (3)), and do not require the assumption \( p(t) \geq 0 \) for a.e. \( t \geq 0 \) in the contrary case \( \int_0^{+\infty} g(s)ds < +\infty \) (corresponding to (4)). Finally, we will show that in both cases \( \int_0^{+\infty} g(s)ds = +\infty \) and \( \int_0^{+\infty} g(s)ds < +\infty \), the Hartman-Wintner theorem can be derived from a certain counterpart of the half-linear extension of the well-known linear Kamenev oscillation criterion given in [2].

References


On the Speed at which Solutions of the Sturm-Liouville Equation Tend to Zero

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We consider the equation

\[-y''(x) + q(x)y(x) = f(x), \quad x \in \mathbb{R}.\]  \hspace{1cm} (1)

For a fixed $p \in [1, \infty)$ and for a correctly solvable equation \((??)\) in $L_p(\mathbb{R})$, we find a positive and continuous function $\alpha_p(x)$ for $x \in \mathbb{R}$ such that we have a sharp by order equality

\[y(x) = o(\alpha_p(x)), \quad |x| \to \infty, \quad \forall y \in \mathcal{D}_p.\]

Here

\[\mathcal{D}_p = \{ y \in L_p(\mathbb{R}) : y, y'\text{loc}(\mathbb{R}), -y'' + qy \in L_p(\mathbb{R}) \}.\]
Consider the first-order delay differential equation

\[ x'(t) + p(t) x(\tau(t)) = 0, \quad t \geq t_0, \quad (1) \]

where \( p, \tau \in C([t_0, \infty), R^+) \), \( \tau(t) \) is nondecreasing, \( \tau(t) < t \) for \( t \geq t_0 \) and \( \lim_{t \to \infty} \tau(t) = \infty \), and the (discrete analogue) difference equation

\[ \Delta x(n) + p(n) x(\tau(n)) = 0, \quad n = 0, 1, 2, ..., \quad (1)' \]

where \( \Delta x(n) = x(n + 1) - x(n) \), \( p(n) \) is a sequence of nonnegative real numbers and \( \tau(n) \) is a nondecreasing sequence of integers such that \( \tau(n) \leq n - 1 \) for all \( n \geq 0 \) and \( \lim_{n \to \infty} \tau(n) = \infty \). Optimal conditions for the oscillation of all solutions to the above equations are presented.
Oscillatory Criteria for Differential Equations with Several Deviating Arguments

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Consider the first-order delay differential equation

$$x'(t) + \sum_{i=1}^{m} p_i(t) x(\tau_i(t)) = 0, \quad t \geq 0, \quad (1)$$

where, for every $i \in \{1, ..., m\}$, $p_i$ is a continuous real-valued function in the interval $[0, \infty)$, and $\tau_i$ is a continuous real-valued function on $[0, \infty)$ such that

$$\tau_i(t) \leq t, \quad t \geq 0, \quad \lim_{t \to \infty} \tau_i(t) = \infty \quad (2)$$

and the (dual) advanced differential equation

$$x'(t) - \sum_{i=1}^{m} p_i(t) x(\sigma_i(t)) = 0, \quad t \geq 1, \quad (3)$$

where, for every $i \in \{1, ..., m\}$, $p_i$ is a continuous real-valued function in the interval $[0, \infty)$, and $\sigma_i$ is a continuous real-valued function on $[0, \infty)$ such that

$$\sigma_i(t) \geq t, \quad t \geq 1 \quad (4)$$

Next, consider the discret analogue differential equations

$$\Delta x(n) + \sum_{i=1}^{m} p_i(n) x(\tau_i(n)) = 0, \quad n \in \mathbb{N}_0 \quad (5)$$

where $2 \leq m \in \mathbb{N}, p_i, 1 \leq i \leq m$, are real sequences and $\{\tau_i(n)\}_{n \in \mathbb{N}_0}, 1 \leq i \leq m$, are sequences of integers such that
\[ \tau_i(n) \leq n - 1, \quad n \in N_0, \quad \lim_{n \to \infty} \tau_i(n) = \infty, \quad 1 \leq i \leq m \quad (6) \]

Here, as usual \( \Delta x(n) = x(n + 1) - x(n) \).
Several oscillation conditions for the above equations are presented.
Variation Formulas of Solution for a Neutral Functional Differential Equation Taking into Account Delay Function Perturbation and the Discontinuous Initial Condition

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For the neutral functional differential equation

\[ \dot{x}(t) = A(t)\dot{x}(\sigma(t)) + f(t, x(t), x(\tau(t))), t \in [t_0, t_1] \]

with the discontinuous initial condition

\[ x(t) = \varphi(t), t < t_0, x(t_0) = x_0 \]

linear representations of the main part of a solution increment (variation formulas) are proved with respect to perturbations of initial moment \( t_0 \), initial function \( \varphi(t) \), initial vector \( x_0 \), delay function \( \tau(t) \) and nonlinear term \( f \) of right-hand side. In the variation formulas, the effects of delay function perturbation and discontinuous initial condition are detected. The variation formula of solution plays the basic role in proving of the necessary conditions of optimality and under sensitivity analysis of mathematical models.

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Almost Periodic Solutions of Impulsive Evolution
Equations

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In Banach space $X$ we consider equation with impulses
\[
\frac{du}{dt} = (A + A_1(t))u + f(t, u), \quad t \neq \tau_j, \\
\Delta u|_{t=\tau_j} = u(\tau_j) - u(\tau_j - 0) = B_j u(\tau_j - 0) + g_j(u(\tau_j - 0)),
\]
where $u : \mathbb{R} \to X$, the sequence of points of impulsive action \(\{\tau_j\}_{j \in \mathbb{Z}}\) has uniformly almost periodic differences, $A$ is a sectorial operator, $X^\alpha = D(A^\alpha), \alpha > 0$, functions $A_1(t) : \mathbb{R} \to L(X^\alpha, X)$ and $f(t, u) : \mathbb{R} \times X^\alpha \to X$ are Bohr almost periodic in $t$, sequences $B_j : \mathbb{Z} \to L(X^\gamma, X^\alpha)$ and $g_j(u) : \mathbb{Z} \times X^\alpha \to X^\alpha$ are almost periodic, $\gamma \geq \alpha$.

We study conditions for existence and stability of piece-wise continuous almost periodic solutions of equation (1), (2).
A Topological Approach to Periodic Oscillations
Related to the Liebau Phenomenon

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In the 1950’s the physician G. Liebau developed some experiments dealing with a valveless pumping phenomenon arising on blood circulation and that has been known for a long time: roughly speaking, Liebau showed experimentally that a periodic compression made on an asymmetric part of a fluid-mechanical model could produce the circulation of the fluid without the necessity of a valve to ensure a preferential direction of the flow [1, 7, 8]. After his pioneering work this effect has been known as the Liebau phenomenon.

In [9] G. Propst, with the aim of contributing to the theoretical understanding of the Liebau phenomenon, presented some differential equations modeling a periodically forced flow through different pipe-tank configurations. He was able to prove the presence of pumping effects in some of them, but the apparently simplest model, the “one pipe-one tank” configuration, skipped his efforts due to a singularity in the corresponding differential equation model, namely

\[
\begin{align*}
  u''(t) + au'(t) &= \frac{1}{\mu} \left( e(t) - b(u^2) - c \right), & t \in [0, T], \\
  u(0) &= u(T), & u'(0) = u'(T),
\end{align*}
\]

being \( a \geq 0, \ b > 1, \ c > 0 \) and \( e(t) \) continuous and \( T \)-periodic on \( \mathbb{R} \).

The singular periodic problem (1) was studied in [2], where general results for the existence and asymptotic stability of positive solutions were obtained using the substitution \( u = x^\mu \), where \( \mu = \frac{1}{b+1} \), which transforms the singular problem (1) into the regular one

\[
\begin{align*}
  x''(t) + ax'(t) &= \frac{e(t)}{\mu} x^{1-2\mu}(t) - \frac{c}{\mu} x^{1-\mu}(t), & t \in [0, T], \\
  x(0) &= x(T), & x'(0) = x'(T).
\end{align*}
\]

In particular, the existence and stability of positive solutions for (2) were then proved by means of the lower and upper solution technique and using tricks analogous to those used in [10].
This presentation is based on the paper [3] where we deal with the existence of positive solutions for the following generalization of problem (2)

\[
\begin{aligned}
& x'''(t) + ax'^\alpha(t) - s(t)x'^\beta(t), \quad t \in [0, T], \\
& x(0) = x(T), \quad x'(0) = x'(T),
\end{aligned}
\]

where we assume

(H0) \( a \geq 0, r, s \in C[0, T], 0 < \alpha < \beta < 1 \).

Our approach is essentially of topological nature: we rewrite problem (3) as an equivalent fixed point problem suitable to be treated by means of the Krasnosel’skiĭ-Guo cone expansion/compression fixed point theorem. Also a careful analysis of the related Green’s function is essential for the proofs. Our main results deal with existence, non-existence and localization criteria for positive solutions of the problem (3). Some corollaries with more ready-to-use results are also addressed. We point out that our results are valid not only for the more general problem (3), but also when applied to the singular model problem (1) we improve previous results of [2].

References


About Differential Inequalities for Nonlocal Boundary Value Problems with Impulsive Delay Equations

I. Volinsky (joint work with A. Domoshnitsky)

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In this talk we propose results about sign-constancy of Green’s functions to impulsive nonlocal boundary value problems in a form of theorems about differential inequalities.

One of the ideas of our approach is to construct Green’s functions of boundary value problems for simple auxiliary differential equations. Careful analysis of these Green’s functions allows us to get decision about the sign of Green’s functions of the main problems in corresponding function spaces.

We adopt this idea to study positivity of Green’s functions in the case of impulsive function differential equations.
On Positivity of Green Functions for a Functional-Differential Equation

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A necessary and sufficient condition of negativity of the Green function of the problem

\[ u'''(x) - \int_{0}^{l} u(s)d_{x}(x, s) = f(x), \quad x \in [0, l], \]

\[ (u(0), u'(0), u(l)) = 0 \]

in terms of spectral radii of two auxiliary problems.
Dirichlet Problem of Delayed Reaction-Diffusion Equations Involving Semi-Infinite Intervals

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We consider a nonlocal delayed reaction-diffusion equation in the half space $\mathbb{R}_+$, which describes the matured population of a single species with a maturation delay living in spatially semi-infinite environment. There is a lack of compactness and lack of spatial symmetry for this system. To overcome the difficulty in describing the global dynamics due to the non-compactness and asymmetry of the spatial domain, we establish a priori estimate for nontrivial solutions after describing the delicate asymptotic properties of the nonlocal delayed effect and the diffusion operator. The estimate enables us to show the repellency of the trivial equilibrium and the existence of heterogeneous steady state in the case of Dirichlet boundary conditions. This further allows us to employ dynamical system arguments to establish the global attractivity of the heterogeneous steady state.