Question 1

In East Prussia, each of the two million citizens owns a “prussak”. It is also known that exactly half of the citizens own 12 “prussaks” each, and the other half own 24 “prussaks” each. Two “prussaks” are called “comrades” if they are owned by the same master (every “prussak” is also his own “comrade”). Find the difference between the average number of “comrades” each “prussak” has and the average number of “prussaks” each citizen of Prussia owns.

Question 2

A pedestrian ran one third of his way at the speed of $v_1 = 12{,}5$ km/h, walked one third of the time at the speed $v_2 = 4{,}5$ km/h, and walked the rest of the way at a speed equal to the average speed of the entire journey. What was the average speed of the pedestrian?

Question 3

Let $f_1(x) = x - x^2$, $f_2(x) = x + x^2$, $g_1(x) = x - x^3$, $g_2(x) = x + x^3$.

Find $\lim\limits_{x \to 0} \dfrac{x - f_1(g_1(f_2(g_2(x))))}{x^3}$.

Question 4

All participants of a certain contest receive cards at the end of the contest. The number of participants is *n*. The contestant who takes first place is given one card and one tenth of all remaining cards, the contestant who takes second place is given two cards and one tenth of all remaining cards, etc., and the contestant who finishes in the last, n^{th}, place is given *n* cards and one tenth of the remaining cards. If after all of the contestants have received their cards no extra cards remain, how many contestants were there?

Question 5

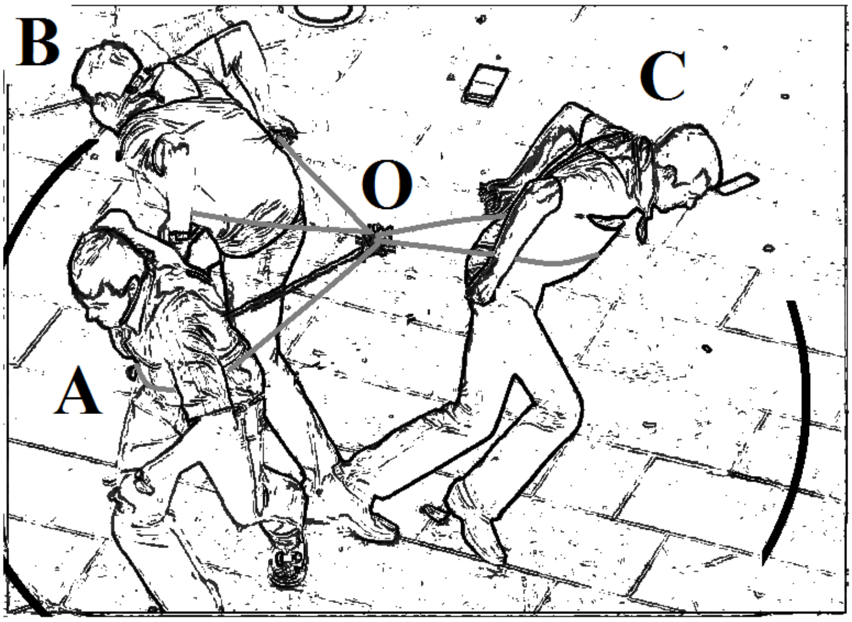
Find the integral $\displaystyle\int_0^{2014} \frac{f(x)}{f(x) + f(2014 - x)}\, dx$.

Question 6

A flea is jumping inside a unit square. Initially it can be anywhere in this square. Every second it chooses a vertex, and gets four times closer to it by jumping towards it. Find the area of the set of points where the flea can be after the fifth jump. (For example, the area of the set of points where the flea can be after the first jump is equal to 1).

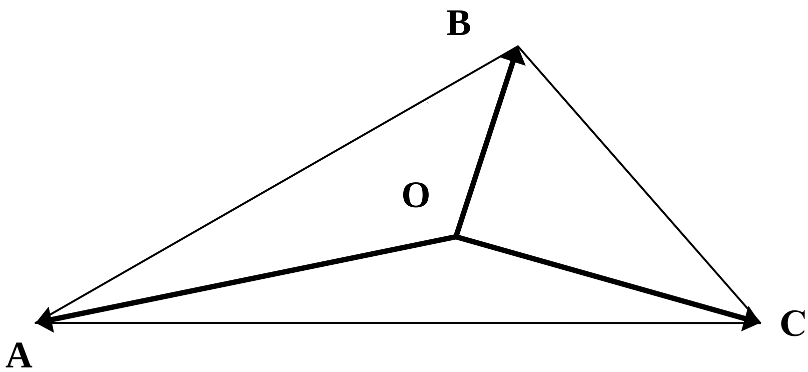
Question 7

Three people are taking part in a tug of war competition (see figure).



The goal of each player is to cross the line drawn in front of him.

Player C is planning to pull his rope with force $\overrightarrow{F_C}$, which is equal in magnitude to the product of the distance from the knot O, which ties the three players' ropes together, to player C, multiplied by the area of the triangle formed by the vectors leading from the knot to his rivals. ($\overrightarrow{F_C} = S_C \cdot \overrightarrow{OC}$, where S_C - is the area of triangle AOB, see figure).



Prove that if the other two players follow the same strategy, none will be able to move ($\overrightarrow{F_C} + \overrightarrow{F_A} + \overrightarrow{F_B} = \vec{0}$).

Question 8

How many different non-singular matrices of order 3 exist, the elements of which are "0" or "1"?

Question 9

Find the integer part of the sum of 4028 terms

$$\sqrt{2014^2+1} + \sqrt{2014^2+2} + \ldots \sqrt{2014^2+2\cdot 2014} = \sum_{k=1}^{2\cdot 2014} \sqrt{2014^2+k}.$$

Question 10

Let $a_1, a_2, \ldots, a_n, a_{n+1}, \ldots, a_{2n-1}, a_{2n}$ all be different positive divisors of the number $N = 2^{2014} + 1$, in ascending order. Find $(a_{n+1} - a_n)$.

Question 11

Victor and Michael are playing a game. Victor chooses a number from 1 to *n*. Michael has to find which number Victor chose by asking questions, such as: "Is the number even?", "Is the number greater than 5?" etc. If the answer to the question is "Yes", Michael has to pay Victor 1$, and if the answer is "No" – 10$. If Michael has 32$, what is the maximal *n* for which he may be able to find the number?

Question 12

What is the maximal number *N* of points which can be positioned on a plane in such a way that every two points will be connected by segments, some of which are red and some –blue, and segments of the same color will not intersect with each other, while segments of different colors will intersect with each other no more than once?