AN ATTEMPT TO IMPROVE THE EVALUATION OF MECHANICAL MATERIAL PROPERTIES FROM THE AXISYMMETRIC TENSILE TEST

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ABSTRACT. This paper deals with new analytical modelling of the classical tensile test with an axisymmetric sample and determining the yield stress of elasto-plastic materials under neck formation. Known for many years, classical Bridgman and Siebel-Davidenkov-Spiridonova's formulae provide certain errors, especially visible in the case of weakly hardening or ideal-plastic materials. A very accurate numerical simulation of the process allowed us to verify analytical results. Based on the numerical simulation, we proposed some modifications to the analytical models. This made us possible to eliminate two questionable assumptions from the classical approach, during derivation the new formula. Comparison of new results with well known formulae shows that for a small range of plastic deformation some progress is reached, however for greater strain further improvement is still advisable.

Introduction.

Tensile testing is an important standard engineering procedure useful to determine important elastic and plastic properties of materials. Since the well known papers of Bridgman (2), Davidenkov-Spiridonova (5), Siebel (11) and Szczepiński (6) have been published, researchers apply those simple formulae to determine the yield stress for elasto-plastic materials in the case of neck creation (for instance paper (1), (3), (4)). Bridgman and Davidenkov-Spiridonova derived their formulae making use of the following assumptions:

- neglecting of the elastic properties at the stage of neck creation,
- application of deformation theory (or plastic flow theory) in Euler's coordinates,
- small deformations (linearity of the strain tensor or the strain rate tensor),
- material incompressibility in the plastic region,
- application of Huber-Mises or Tresca yield criteria,
- equality of the circumferential strain (circumferential strain rate) to the radial strain (radial strain rate),
- constancy of the yield stress k along the minimum cross section at every increment,
- utilisation of specific formulae for the radius of curvature of the longitudinal stress trajectory, $\rho = \rho(r)$, derived by Bridgman and assumed by Davidenkov-Spiridonova in forms:

$$\rho = \frac{(a^2 + 2aR - r^2)}{2r}, \rho = \frac{aR}{r}, \quad (1)$$

where $0 \leq r \leq a$ and R is the external neck radius (see Fig. 1) at point $r = a$ and $z = 0$.

Figure 1. Geometry of deformed sample.

As a result, the formulae for the average longitudinal stress across the minimum cross section for Bridgman, Siebel-Davidenkov-Spiridonova and Szczepiński were found respectively in the following forms (see papers (2), (5) and (6)):

$$\frac{\bar{\sigma}_z}{k} = \left(1 + \frac{2R}{a}\right) \ln\left(1 + \frac{a}{2R}\right), \quad \frac{\bar{\sigma}_z}{k} = 1 + \frac{a}{4R}, \quad \frac{\bar{\sigma}_z}{k} = \frac{2R}{a} \left[\exp\left(\frac{a}{2R}\right) - 1 \right]. \quad (2)$$

Let us note that the same result as Davidenkov-Spiridonova has been earlier obtained by Siebel and Schwaigerer (11). However, from the best authors' knowledge, this paper was published only in Germany and is not well known. Also Szczepiński formula (2)₃ can be hardly found in engineering literature.

In paper (7) we brought attention that errors connected with application of Bridgman and Davidenkov-Spiridonova formulae to determine the flow curve in the case of ideal plasticity can reach even 10%. By means of simple extension of one of the classical assumptions, we also proposed there a new formula which was slightly better than all of the classical ones. In this paper, we continue our research and derive a new formulae for $\bar{\sigma}_z/k$, based on another set of assumptions than in paper (7).

Discussion on Classical Formulae

Among assumptions utilised during derivation the classical formulae in papers (2), (5), (6) and (11), three are more questionable than the others. In paper (7) we showed that assumptions about equality of the circumferential strain to the radial strain and constancy of the yield stress along the minimum cross section at each increment maybe not as good as one can expect. In this section we are going to discuss another classical assumption concerning the shape of formulae for the radius of curvature of the longitudinal stress trajectory. We will show that the choice of the formula for the curvature $\rho(r)$ from the last assumption is significant. These formulae can be generalised in the form:

$$\rho(r) = \frac{Ra}{rG'(r^2)}, \quad (3)$$

where properties of the function $G(t)$ have been discussed in paper (7). Particularly, another formula than those mentioned in (1) has been proposed:

$$G'(t) = \beta + (a/\sqrt{t})^\alpha (1 - \beta). \quad (4)$$

Finally utilising the equation (3)-(4) and adopting remaining Bridgman assumptions, the formula for the yield stress takes the form:

$$\frac{\bar{\sigma}_z}{k} = 1 + \frac{a}{4R} + \frac{a(1-\beta)\alpha}{4R(4-\alpha)}. \quad (5)$$

Let us note here that for $\alpha = 0$ or $\beta = 1$ Siebel-Davidenkov-Spiridonova curve is obtainable automatically. In Fig. 2 the numerically calculated (from FEM) ratio $\bar{\sigma}_z/\bar{k}$ is presented by points for the different flow curves at some stage of deformations.

Figure 2. Ratio $\bar{\sigma}_z/\bar{k}$ as a function of a/R obtained from FEM simulation and its approximations by Bridgman, Siebel-Davidenkov-Spiridonova and Szczepiński formulae.

Additionally, curves corresponding to Bridgman, Siebel-Davidenkov-Spiridonova and Szczepiński formulae are given. As can be seen from Fig. 2 Szczepiński formulae, derived in a similar way to the classical ones, reveals slightly better approximation than all aforementioned classical formulae. We also approximate numerical results marked by the dots in the Fig. 2 by the new formulae (5) to find optimal values of parameters α , β . In the case of the linear and the nonlinear hardening both α and β are near to 1, so that the curve is close to the Siebel-Davidenkov-Spiridonova one. But in the case of ideal plasticity β is near to 0 and α is in the same range as for linear and nonlinear hardening. As a result, we can just choose some intermediate values: $\alpha = 0.5$ and $\beta = 0.5$, which are not any optimum for particular materials but this approximation is anyway good enough for all cases of materials under consideration. Additionally to the above mentioned curves in Fig. 2, we show the curve drawn due to formula (5) with these parameters. Let us mention here that points $\bar{\sigma}_z/\bar{k}$ which correspond to the ideal plasticity do not follow a straight line near the origin of the coordinate system. This means that other ideas than the classical ones from (2), (5) should be employed and additionally other parameters should be taken into account to improve the classical formulae. For example, it maybe important to choose the curvature (3) in a more general form $\rho(r) = F(r, a/R, a/a_0)$

as a function of two measured parameters: a/R and a/a_0 . The first one just has been taken into account in (1)-(3), while the second traces the level of the plastic deformation of the sample (see Fig. 1).

A NEW FORMULA DERIVATION IN EULER'S COORDINATES

Main Assumptions

1. In our considerations, we consistently apply the deformation theory of plasticity. The reason for this is that strain is uniform and monotonic up to the moment of the neck appearance. Simultaneously, we can expect the error connected with application of the deformation theory will be small in comparison with the results obtained on the basis of the more general plastic flow theory when the neck is not deep.
2. We assume displacements along the r and z axes, measured in Euler coordinates in the forms:

$$u_r = B_1(z)r + B_3(z)r^3, \quad u_z = A_1(r)z + A_3(r)z^3 + A_5(r)z^5. \quad (6)$$

The choice of functions u_r , u_z is limited by the fact that we consider only small surroundings of the minimum cross-section. Let us note that, up to the time of neck appearance, the simpler representations: $u_r = B_1 r$ and $u_z = A_1 z$ are valid with constants A_1 , B_1 . Let us also draw attention to the fact that functions u_r , u_z should be smooth enough and are represented by even and odd functions, respectively. According to deformation theory of plasticity (see, for example, the classic monograph (9)) we can write:

$$\varepsilon_{rz} = \lambda \tau_{rz}, \quad \varepsilon_r = \lambda(\sigma_r - \sigma_0), \quad \varepsilon_z = \lambda(\sigma_z - \sigma_0), \quad \varepsilon_\theta = \lambda(\sigma_\theta - \sigma_0). \quad (7)$$

Here $\sigma_0 = (\sigma_r + \sigma_z + \sigma_\theta)/3$ is the hydrostatic pressure and λ is unknown function which has to be found at each point and each increment from the Huber-Mises condition. Additionally, we apply the incompressibility condition which takes form: $\varepsilon_r + \varepsilon_z + \varepsilon_\theta = 0$.

Natural boundary conditions in this issue can be collected as follows:

$$\tau_{rz}|_{r=0} = 0, \quad \tau_{rz}|_{z=0} = 0, \quad \sigma^{(n)}|_{r=\psi(z)} = 0, \quad u_z(r,0) = 0, \quad u_r(0,z) = 0. \quad (8)$$

During an experiment, we can determine the values: radius of the sample in the minimal cross section a , function describing the neck contour $\psi(z)$ and applied force F , measured at the ends of the sample. From information about the function $\psi(z)$ one can calculate the radius of curvature R in the minimal cross section. Gained values lead to the following relationships:

$$u_r(a,0) = a - a_0, \quad \rho(a,0) = R, \quad \bar{\sigma}_z|_{z=0} = \frac{F}{\pi a^2}. \quad (9)$$

The relation, which connects positions of the points on the free surface with their initial coordinates, has the form:

$$r = a_0 + u_r(r,z) \quad \text{at} \quad r = \psi(z). \quad (10)$$

Last equations (9)-(10) will be used in the following as additional boundary conditions and enable us to determine polynomials $B_j(z)$ and $A_i(r)$.

Derivation of the Coefficients Utilised in Displacement Functions

Conditions $(8)_4$, $(8)_5$ are always satisfied for the sake of equation (6). On the other hand, conditions $(8)_1$, $(8)_2$ can be replaced, according to $(7)_1$, by:

$$\varepsilon_{rz}|_{r=0} = 0, \quad \varepsilon_{rz}|_{z=0} = 0, \quad (11)$$

which leads together with the incompressibility condition to:

$$u_z = -\left(2b_{10} + 4b_{30} \frac{r^2}{a^2}\right)z - \left(\frac{2}{3}b_{12} + \frac{4}{3}b_{32} \frac{r^2}{a^2}\right)z^3 - \left(\frac{2}{5}b_{14} + \frac{4}{5}b_{34} \frac{r^2}{a^2}\right)z^5, \quad (12)$$

$$u_r = (b_{10} + b_{12}z^2 + b_{14}z^4)r + (b_{30} + b_{32}z^2 + b_{34}z^4)\frac{r^3}{a^2}, \quad (13)$$

$$\varepsilon_z = -2b_{10} - 4b_{30} \frac{r^2}{a^2} - \left(2b_{12} + 4b_{32} \frac{r^2}{a^2}\right)z^2 - \left(2b_{14} + 4b_{34} \frac{r^2}{a^2}\right)z^4, \quad (14)$$

$$\varepsilon_r = b_{10} + b_{12}z^2 + b_{14}z^4 + 3(b_{30} + b_{32}z^2 + b_{34}z^4)\frac{r^2}{a^2}, \quad (15)$$

$$\varepsilon_\theta = b_{10} + b_{12}z^2 + b_{14}z^4 + (b_{30} + b_{32}z^2 + b_{34}z^4)\frac{r^2}{a^2}, \quad (16)$$

$$\varepsilon_{rz} = \left(b_{12}r + b_{32} \frac{r^3}{a^2} - 4b_{30} \frac{r}{a^2}\right)z + \left(2b_{14}r + 2b_{34} \frac{r^3}{a^2} - \frac{4}{3}b_{32} \frac{r}{a^2}\right)z^3 - \frac{4}{5}b_{34} \frac{r}{a^2} z^5, \quad (17)$$

where constants b_{10} , b_{12} , b_{14} , b_{30} , b_{32} and b_{34} have to be defined to satisfy all conditions. Making use of natural constraint $r'(0) = 0$ and, hence, $r'(z)$ has slight value in a small surrounding of the minimum cross section ($z \ll 1$), the differential equation describing the trajectory of the main stress is of the form:

$$\frac{\varepsilon_{rz}}{\varepsilon_z - \varepsilon_r} = \frac{r'}{1 - (r')^2} = r' + (r')^3 + O(r')^5, \quad r'(z) \rightarrow 0, \quad z \rightarrow 0. \quad (18)$$

We search for the derivative of the longitudinal stress trajectory in the form:

$$r'(z) = zc_1(r) + c_2(r)z^3 + O(z^5), \quad z \rightarrow 0, \quad (19)$$

where $c_1(r)$, $c_2(r)$ are determined from the estimate:

$$\frac{\varepsilon_{rz}}{\varepsilon_z - \varepsilon_r} = z[c_1(r) + c_2(r)z^2] + O(z^5), \quad z \rightarrow 0, \quad (20)$$

with taking into account (14), (15) and (17). As a result, functions $c_1(r)$, $c_2(r)$ depend on the same unknown constants b_{ij} .

The curvature radius of the main stress trajectories will be then calculated from formula:

$$\rho = \frac{(1+r'(z)^2)^{\frac{3}{2}}}{|r''(z)|} \Big|_{z=0} = \frac{1}{|c_1(r)|} = \frac{1}{r} \left| \frac{3b_{10}a^2 + 7b_{30}a^2}{b_{12}a^2 + b_{32}a^2 - 4b_{30}} \right|. \quad (21)$$

Making use of condition (8)₃ and an additional assumption that the neck is not deep, we finally arrive at:

$$\frac{\varepsilon_{rz}}{\varepsilon_z - \varepsilon_r} \Big|_z = \frac{\psi'}{1 - (\psi')^2} \sim \psi' (1 + (\psi')^2), \quad \psi' \rightarrow 0, \quad z \rightarrow 0, \quad (22)$$

where $r = \psi(z)$ is the aforementioned function describing the neck contour (see Fig. 1). It can be approximated from the numerical dates in the form: $\psi(z) = a + d_1 z^2 + d_2 z^4 + O(z^6)$, where constants d_1 and d_2 change from increment to increment. Replacing both sides of relation (18) and comparing the coefficients at appropriate power to the variable z^1 and z^3 , we obtain:

$$\frac{a \left(b_{12} + b_{32} - 4 \frac{b_{30}}{a^2} \right)}{(-3b_{10} - 7b_{30})} = 2d_1, \quad (23)$$

$$\frac{a}{(-3b_{10} - 7b_{30})} \left[2b_{14} + 2b_{34} - \frac{4}{3} \frac{b_{32}}{a^2} - \frac{3b_{12} + 7b_{32}}{3b_{10} + 7b_{30}} \left(b_{12} + b_{32} - 4 \frac{b_{30}}{a^2} \right) \right] = 8d_1^3 + 4d_2. \quad (24)$$

Let us note that both group of equations (18)-(20) and (22)-(24) represent in fact the same type conditions but the first group are valid in the neck interior while the second ones – on the free surface.

We still have to satisfy two conditions (9)₁, (9)₂, from which we finally obtain:

$$\left| \frac{b_{12}a^2 + b_{32}a^2 - 4b_{30}}{-3b_{10}a^2 - 7b_{30}a^2} \right| = \frac{a}{R}, \quad b_{10} + b_{30} = 1 - \frac{a_0}{a}. \quad (25)$$

Making use of condition (10) and replacing right-hand side with relation (13) and also known function describing the neck contour, we receive:

$$\frac{a}{R} = \frac{2a^2(b_{12} + b_{32})}{1 - b_{10} - 3b_{30}}, \quad d_2 = \frac{a(b_{14} + b_{34}) + \frac{1}{2R} \left(b_{12} + \frac{3b_{30}}{2aR} + 3b_{32} \right)}{(1 - b_{10} - 3b_{30})}, \quad (26)$$

for appropriate power to the variable z^1 and z^3 . To calculate the value of six unknown coefficients b_{ij} utilised in displacement functions we dispose of only five equations. For this reason we decided to restrict ourselves to small neck formation when the neck is not deep. This makes us possible to apply asymptotic techniques.

Asymptotical Analysis for Small Neck

Now we will derive the relationship $\bar{\sigma}_z / \bar{k}$ for small values of the ratio a/R indicated later by δ . After some algebra in relations (23)-(26) constants b_{10} , b_{12} , b_{30} , b_{32} , can be found in the forms:

$$b_{10} = \frac{11\delta\Delta + \delta - 8\Delta}{2(-4 + 3\delta)}, \quad b_{30} = \frac{\delta(-5\Delta - 1)}{2(-4 + 3\delta)}, \quad \Delta = 1 - \frac{a_0}{a}, \quad b_{12} = \frac{\delta^4(-12\Delta - 24)}{8a^2(-4 + 3\delta)(3\delta - 2)} - \frac{\delta^3(51\Delta - 105) + \delta^2(80\Delta - 128) + \delta(32 - 32\Delta) + \delta a^3(48d_2\Delta + 96d_2) - a^3(96d_2 - 96d_2\Delta)}{8a^2(-4 + 3\delta)(3\delta - 2)}$$

$$b_{32} = \frac{-3[16\delta^2(\Delta - 1) - 4\delta^4(\Delta + 2) - \delta^3(25\Delta - 19) + 16d_2a^3(\delta\Delta + 2\delta - 2 + 2\Delta)]}{8a^2(-18\delta + 9\delta^2 + 8)}. \quad (27)$$

We did not present here the value of coefficients b_{14} and b_{34} because they would not appear in equation of equilibrium (29). Our further considerations will be made for the minimum cross section, i.e. $z = 0$. In order to obtain the stress distribution along the minimum cross section we will integrate the equilibrium equation for an axisymmetric problem, which has the form:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{1}{r}(\sigma_r - \sigma_\theta) = 0. \quad (28)$$

Being based on equations of deformation theory of plasticity (7) and describing strains (14)-(17), we are able to rewrite (28) in the equivalent form

$$\frac{\partial \sigma_r}{\partial r} - \frac{r(b_{12}a^2 + b_{32}r^2 - 2b_{30})}{3b_{10}a^2 + 7b_{30}r^2}(\sigma_z - \sigma_r) = 0, \quad (29)$$

with boundary condition of ordinary differential equation:

$$\sigma_r|_{r=a} = k. \quad (30)$$

Inserting the additional parameter:

$$\beta(r) = \frac{\varepsilon_r - \varepsilon_\theta}{\varepsilon_z - \varepsilon_r} \Big|_{z=0} = \frac{2b_{30}r^2}{-3b_{10}a^2 - 7b_{30}r^2},$$

one can conclude from (7) and Huber-Mises yield condition that

$$\sigma_z - \sigma_r = \frac{k}{\sqrt{1 + \beta^2(r) + \beta(r)}}. \quad (31)$$

Integrating equation (29), with using equation (31) and the boundary condition (30), we obtain:

$$\sigma_r = \int_a^r \frac{b_{12}a^2 + b_{32}t^2 - 2b_{30}}{3b_{10}a^2 + 7b_{30}t^2} \frac{kdt}{\sqrt{1 + \beta + \beta^2}}. \quad (32)$$

Making use once more of equations (31), we finally receive the longitudinal stress distribution in the neck as:

$$\sigma_z - \frac{k}{\sqrt{1 + \beta^2 + \beta}} = \int_a^r \frac{b_{12}a^2 + b_{32}t^2 - 2b_{30}}{3b_{10}a^2 + 7b_{30}t^2} \frac{kdt}{\sqrt{1 + \beta + \beta^2}}. \quad (33)$$

Let us determine now the average longitudinal stress across the minimum section from equation (33), with an accuracy to $O(\delta^2)$ at $\delta \rightarrow 0$:

$$\bar{\sigma}_z = \frac{2}{a^2} \int_0^a t \sigma_z(t) dt = \bar{k} \int_0^1 \left[1 - \frac{a^2 t}{2(3b_{10} + 7b_{30}t)} \left(b_{12} - 4 \frac{b_{30}}{a^2} + b_{32}t \right) \right] dt. \quad (34)$$

Here we utilized the fact that $\beta(r) = O(\delta)$ and took into consideration the additional assumption verified with numerical simulation:

$$k(r,0) = \bar{k} + \Delta k(r), \quad \Delta k(r) \sim \delta, \quad \delta \rightarrow 0. \quad (35)$$

Finally, we receive with the mentioned accuracy of $O(\delta^2)$:

$$\frac{\bar{\sigma}_z}{\bar{k}} = 1 - \frac{b_{32}a^2}{28b_{30}} - \frac{b_{12}a^2}{14b_{30}} + \frac{2}{7} + \frac{3b_{10}b_{32}a^2}{98b_{30}^2} + \frac{3b_{10}}{7b_{30}} \left(\frac{b_{12}a^2}{14b_{30}} - \frac{2}{7} - \frac{3b_{10}b_{32}a^2}{98b_{30}^2} \right) \ln \left(1 + \frac{7b_{30}}{3b_{10}} \right) \quad (36)$$

with the constants from (27). It is worth noticing that the asymptotical behaviour of the obtained solution absolutely coincides with Bridgman, Siebel-Davidenkov-Spiridonova and Szczepiński formulae when $\Delta = 0$ and $\delta \rightarrow 0$.

In paper (8) we have presented another new formula for the normalised axial stress derived within Lagrange's coordinate approach.

Comparison of the Obtained Formula with Numerical Simulations and Discussions

In this chapter we will describe the numerical simulation which is carried out to verify the new formula. We consider an axisymmetric sample so only one-half of the length of the tensile specimen was modelled where the axial coordinate z ranged from 0 to 30 mm and the radial coordinate r ranged from 0 to 5 mm. All nodes along the $z = 30$ axis would continue to have their z -displacements increased by a constant increment. A very dense mesh, consisted of 64200 elements and concentrated in the surroundings of the minimum cross-section, was created. The material of the considered specimen was assumed to be elasto-plastic with Young's modulus of 210000 MPa, Poisson's ratio of 0.3 and the initial yield strength of 200 MPa. We carried out the simulations for different model flow curves, i.e. nonlinear hardening with $k(\bar{\epsilon}^p) = 100 + 100(1 + 14.24775\bar{\epsilon}^p)^{0.5}$, linear hardening with a plastic modulus of 150 MPa and the ideal plasticity model. By means of appropriate indicated marker, the relations between $\bar{\sigma}_z/\bar{k}$ and a/R obtained from numerical simulations are shown in Fig. 3 (Lagrange's coordinate approach) and Fig. 4 (Euler coordinate approach) together with curves obtained from the new formula (36) and drawn due the classical Bridgman and Siebel-Davidenkov-Spiridonova and Szczepiński formulae.

Figure 3. Normalised axial stress obtained from FEM (markers), results corresponding to the formula received in the previous paper (8) under Lagrange's coordinate approach (dashed lines). Bridgman, Siebel-Davidenkov-Spiridonova and Szczepiński curves are drawn for comparison.

Note here that the new formula (36) takes into account additionally the initial radius of sample. As a result, obtained curves differ for every material in contrast to the classical formulae which gives the same unique results. The new formulae derived by us with the same technique but under different, Euler and Lagrange, approaches give better results in comparison with Szczepiński formula (the best from the classical ones). However, the new formula (36) derived under Euler approach has a wider range of applicability and reveals better accuracy. Comparing it with Szczepiński one the accuracy was 0.1% and 1.1% respectively for a linear hardening, whereas for nonlinear hardening and ideal plasticity we have: 0.3% via 1.7% and 0.5% via 3.2%. Unfortunately, these results are valid only for small plastic deformation (see Fig. 4), while according to paper (10) most of materials fracture at ratio a/R near 0.7. Our future research is aimed to expand the applicability range of the new formula.

Figure 4. Normalised axial stress obtained from FEM (markers) and the new formula (36) (dashed lines). Bridgman, Siebel-Davidenkov-Spiridonova and Szczepiński curves are drawn for comparison.

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