Size effect in behavior of lightly reinforced concrete beams

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Abstract
The aim of this work is to assess the effect of size (depth of section) in the behavior of reinforced concrete beams. A computation scheme and stress-strain relations are proposed for analysis of reinforced concrete sections with regard to this effect for the case of short-term loading. The proposed model includes two distinct stress-strain relations for tensile concrete. One relation represents the behavior of concrete in axially-loaded reinforced concrete members (the tension stiffening effect). It is intended for tensile concrete close to the reinforcing bars. The other relation is intended to represent the behavior of plain concrete in flexural tension. Reported in literature experimental data, in particular, data obtained in tests on beams with depth of section from 20 to 80 cm, are used for calibration of parameters in this work. Analyses based on the proposed model show that the shape of moment-curvature diagrams for beams with low steel ratios is markedly influenced by the size effect.

Keywords: beams, modeling, reinforced concrete, tensile properties.

Introduction
The unsatisfactory performance of modern reinforced concrete buildings in the 2010/2011 Canterbury, New Zealand, earthquakes is attributed, among other factors, to the unexpected by the designers rocking, rather than flexural, mode of behavior of laterally loaded lightly reinforced concrete walls [1]. In principle, the transition from the ductile flexural behavior, typical for relatively small laboratory specimens, to brittle rocking behavior of reinforced concrete walls in multi-storey buildings can be identified as a size effect. However, there is no consensus with regard to this effect even in behavior of reinforced concrete beams – the most common structural elements. The size effect in their behavior is recognized by the fib MC [2] and not recognized by the ACI code [3].

The aim of this work is to consider this issue and re-examine available test data.

Two different approaches to modeling of the behavior of reinforced concrete beams are found in literature. According to one approach, the cracked concrete is modeled as a gradually softening material and the averaged stress distribution in a section is as shown in Fig. 1a. Calculations based on this approach in different versions (see e.g. [4-6]) yield results (curvatures, deflections) in reasonable agreement with test data for members with depth of section 40 cm or less.

According to the other approach (see e.g. [2, 7, 8]), which is based, in particular, on experimental studies [9, 10] of large reinforced concrete beams, the cracked concrete bears the tensile stress within some area close to reinforcing bars (the tension stiffening effect) but behaves in a brittle manner in the remain of tensile concrete area; the averaged stress distribution is as shown in Fig. 1b.

In this paper, the above approaches are combined and a model of concrete behavior is presented, which is equally suitable for analyses of reinforced concrete beams of any size.

The paper is organized as follows:
1. A computation scheme is given for nonlinear analysis of reinforced concrete sections in flexure for the case of short-term loading.

2. Using this scheme and experimental moment-curvature diagrams for reinforced concrete beams with depth of section from 20 to 80 cm, assessment is made of the effective (averaged) stress-strain relations of plain concrete in flexural tension.

3. The effect of size on moment-curvature diagrams is illustrated for sections with different steel ratios.

Analysis of reinforced concrete section in flexure

Consider a reinforced concrete section (Fig. 2) acted upon by the bending moment \( M \). As usual, cracks in the beam are “smeared” and linear distribution of the strain, \( \varepsilon \), over the section is assumed

\[ \varepsilon = \varepsilon_0 + \phi x. \]  

(1)

Here \( \phi \) is the curvature, \( \varepsilon_0 \) – the strain of the extreme compressive fiber (at which the x-axis origin is located). The concrete section comprises three zones: \( A_{cc} \) - compressive concrete with stress-strain relation \( \sigma_c=\sigma_{cc}(\varepsilon) \), \( A_{rct} \) – reinforced tensile concrete with averaged stress-strain relation \( \sigma_c=\sigma_{rct}(\varepsilon) \), \( A_{pct} \) - plain tensile concrete with averaged stress-strain relation \( \sigma_c=\sigma_{pct}(\varepsilon) \).

Following [11], the depth \( d_{rct}=2(H-D) \) of zone \( A_{rct} \) is adopted, where \( H \), \( D \) are the total and effective depths of the section, respectively.

The solution \((\varepsilon_0, \phi)\) of the equilibrium equations

\[ \int_0^H \sigma_s b dx + \sigma_s A_s D = M, \]  

(2a)

\[ \int_0^H \sigma_s b dx + \sigma_s A_s = 0, \]  

(2b)

where \( b=b(x) \), \( \sigma_s, A_s \) are the width of the section and the steel stress and area respectively, is found using the following iteration scheme

\[ \left( \int_0^H E_c^{(i)} b dx + E_s A_s D \right) \varepsilon_0^{(i+1)} + \left( \int_0^H E_c^{(i)} b x^2 dx + E_s A_s D^2 \right) \phi^{(i+1)} = M, \]  

(3a)

\[ \left( \int_0^H E_c^{(i)} b dx + E_s A_s \right) \varepsilon_0^{(i+1)} + \left( \int_0^H E_c^{(i)} b x dx + E_s A_s D \right) \phi^{(i+1)} = 0, \]  

(3b)

\[ i = 0, 1, 2, \ldots \]  

(3c)

where

\[ E_c^{(i)} = \begin{cases} \sigma_{cc}(\varepsilon^{(i)}) / \varepsilon^{(i)} & \text{if } \varepsilon^{(i)} < 0, \\ E_{c0} & \text{if } \varepsilon^{(i)} = 0, \\ \sigma_{rct}(\varepsilon^{(i)}) / \varepsilon^{(i)} & \text{if } \varepsilon^{(i)} > 0 \text{ and } x < H - d_{rct}, \\ \sigma_{pct}(\varepsilon^{(i)}) / \varepsilon^{(i)} & \text{if } \varepsilon^{(i)} > 0 \text{ and } x \geq H - d_{rct}. \end{cases} \]  

(4)

\( E_{c0} \) - initial Young’s modulus of the concrete, \( E_s = 200 \text{ GPa} \) - Young’s modulus of the steel,
\[ \varepsilon^{(i)} = \varepsilon^{(i)}_0 + \phi^{(i)}x, \quad (5a) \]
\[ \varepsilon^{(0)}_0 = 0, \quad \phi^{(0)} = 0, \quad (5b) \]
with the convergence criterion
\[ |\varepsilon^{(i+1)}_0 - \varepsilon^{(i)}_0| \leq 0.001|\varepsilon^{(i)}_0|, \quad |\phi^{(i+1)} - \phi^{(i)}| \leq 0.001|\phi^{(i)}|. \quad (6) \]

**Stress-strain relations of concrete**

For the tensile concrete in zone A_{pct}, the stress-strain relation can be taken in form similar to that adopted in Ref. [12]

\[ \sigma_{pct}(\varepsilon) = \begin{cases} E_{c0} \varepsilon & \text{if } \varepsilon \leq \varepsilon_{f\ell} = f_{ct} / E_{c0}, \\ f_{ct} \phi(\varepsilon, \varepsilon_{f\ell}, \gamma) & \text{if } \varepsilon > \varepsilon_{f\ell}, \end{cases} \quad (7) \]

where \( f_{ct} \) is the tensile strength of the concrete and

\[ \phi(\varepsilon, \varepsilon_{f\ell}, \gamma) = \begin{cases} \gamma(\varepsilon / \varepsilon_{f}) & \text{if } \gamma > 1, \\ 1 + (\gamma - 1)(\varepsilon / \varepsilon_{f})^{\gamma/(\gamma-1)} & \text{if } \gamma = 1. \end{cases} \quad (8) \]

Here \( \gamma \) is a parameter that depends on the steepness of the descending branch of the stress-strain diagram (see Fig.3).

Fitting moment-curvature diagrams obtained with use of the above computation scheme to experimental data presented in Figs. 4-6, that is, finding for each group of data such a value \( \gamma \) that

\[ \phi_{\text{fit}} \left\{ \int_{0}^{\phi_{\text{max}}} M_{th}(\gamma) - M_{exp} \right\} d\phi = \min, \quad (9) \]

where \( M_{th}(\gamma) = M_{th}(\phi_{\gamma}) \) is the theoretical bending moment corresponding to the specific value of \( \gamma \), \( M_{exp} = M_{exp}(\phi) \) - the experimental bending moment and \( \phi_{\text{max}} \) - the curvature in the cracking stage corresponding to the end point of the diagram, we obtain (see Fig. 7)

\[ \gamma = 3.2 - 0.7(H / H_0) \geq 1, \quad H_0 = 20cm, \quad (10) \]

Other relations adopted for constructing the diagrams in Figs. 4-6 are as follows:
- for the tensile concrete in zone A_{ret}, as per Refs. [16, 17]
  \[ \sigma_{ret}(\varepsilon) = \begin{cases} E_{c0} \varepsilon & \text{if } \varepsilon \leq \varepsilon_{f\ell}, \\ \phi(\varepsilon, \varepsilon_{f\ell}, 2.5) & \text{if } \varepsilon > \varepsilon_{f\ell}, \end{cases} \quad (11) \]
- for the compressive concrete, as per Ref. [18]
  \[ \sigma_{cc}(\varepsilon) = f_c \phi(\varepsilon, \varepsilon_{f\ell}, E_{c0} \varepsilon_{f\ell} / f_c), \quad (12) \]
  where \( f_c \approx 0.8f_{cu} \) is the compressive strength of the concrete, \( f_{cu} \) – the cube strength and \( \varepsilon_{fc} \) – the strain corresponding to \( f_c \);
- for \( E_{c0} \), as per Ref.[19]
  \[ E_{c0} = \frac{51500(f_c / f_{c0})}{2 + (f_c / f_{c0})} \quad [\text{MPa}], \quad (13) \]
  where \( f_{c0} = 10\text{MPa}; \)
- for \( \varepsilon_{fc} \), as per Refs. [20,21]
\[ \varepsilon_{fc} = -0.00156 - 0.00014 \left( \frac{f_c}{f_{cr}} \right) \]  \hspace{1cm} (14)

- for \( f_{cr} \), as per Ref. [22]

\[ f_{cr} = 0.52 \left( -f_{ct} \right)^{2/3} \text{ [kgf/cm}^2] \]  \hspace{1cm} (15)

**Effect of size on moment-curvature diagrams**

See the diagrams in Fig. 8, constructed using the computation scheme and stress-strain relations given above.

**Conclusion**

Unlike beams with steel ratios 0.01 and more, the behavior of lightly reinforced concrete beams is markedly influenced by the size effect (depth of section). A computation scheme and stress-strain relations are given for analysis of reinforced concrete sections with regard to this effect for the case of short-term loading.

**References**


Figures and figure captions

Fig. 1. Effective areas (hatched) of tensile concrete and concrete stress distributions ($f_{ct}$ – concrete tensile strength) in reinforced concrete section according to concrete models as a softening material (a) and as a tensile-stiffening material (b)
Fig. 2. Reinforced concrete section; strain distribution

Fig. 3. Averaged stress-strain diagrams for concrete in flexural tension
Fig. 4. Estimates of $\gamma$ for $H=20$ cm and $H=40$ cm; moment-curvature diagrams fitted to experimental data from Ref. [13] ($f_{cu}$ – concrete cube strength, $\rho=A_s/(bD)$)
Fig. 5. Estimates of $\gamma$ for $H=62.5$ cm; moment-curvature diagrams fitted to experimental data from Ref. [14]

Fig. 6. Estimates of $\gamma$ for $H=80$ cm; moment-curvature diagrams fitted to experimental data from Ref. [15]; cross-section of beams (dimensions in cm)
Fig. 7. $\gamma$ versus $H$

Fig. 8. Comparison of moment-curvature diagrams for reinforced concrete rectangular sections of different depth ($f_c=-30$ MPa, $D=0.9H$)