THz COHERENT SPONTANEOUS UNDULATOR EMISSION FROM SHORT ELECTRON BUNCHES IN NEGATIVE MASS REGIME

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Abstract

It is known that Negative Mass Instability (NMI) can take place not only in cyclic accelerators but also in an undulator combined with a strong over-resonance field of a focusing magnet. In the undulator case, the effect occurs due to a specific dependence, $\frac{\partial\beta_\parallel}{\partial\gamma} < 0$, of electron longitudinal velocity on the particle energy $E = m_e c^2 \gamma$. The corresponding decrease or increase in the electron longitudinal velocity with an increase or decrease in the electron energy due to an additional particle acceleration in the Coulomb field of an electron density inhomogeneity, can lead to attraction of the electron to an inhomogeneity of the charge density and growth of the latter. This grows of an inhomogeneity originated from the initial shape of the bunch was numerically demonstrated for sub picosecond, highly charged electron bunches, or for periodically pre-modulated trains of such bunches. For these both types of the bunch shape, NMI enables retaining of their longitudinal profile during a long-distance bunch propagation through the combined magnetic field of the system. A remarkable quasi-periodic self-modulation was found to be excited and developed, due to the radiation instability and thanks to NMI, during a propagation of relatively long non-modulated electron bunches. Long-lived keeping of the drive bunch profile gives rise to strong coherent spontaneous emission in all these configurations which are characterized by the radiation energy extraction of 10-20%, what suggests realization of compact and powerful sources of intense and narrow-band THz radiation.

1. Introduction

Short electron bunches formed in laser-driven photo-injectors [1-3] are attractive for generation of coherent spontaneous terahertz radiation (see e.g. [4-13]). However, energy and spectral capabilities of terahertz sources driven by such bunches are significantly limited due to strong Coulomb repulsion of particles, which leads to fast expansion of bunches and loss of the radiation coherency. This expansion can partially be mitigated [14] in a field of helical undulator combined with a very strong (over-resonance) guiding field, when the conditions for non-isochronous longitudinal electron oscillations [15-17] are fulfilled. In this case, development of the Negative Mass Instability (NMI) can lead to formation of a long-living electron “core” with a sufficiently small longitudinal size [14, 18]. The Negative Mass Instability in undulators [15-17] is similar to the classical NMI first discovered in cyclic accelerators [19, 20] and then studied also for Cyclotron Resonance Masers [21, 22]. The NMI in undulators occurs due to a specific dependence, $\frac{\partial\beta_\parallel}{\partial\gamma} < 0$, of electron longitudinal velocity on the particle energy $E = m_e c^2 \gamma$. The corresponding decrease or increase in the electron longitudinal velocity with an increase or decrease in the

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electron energy due to additional particle acceleration in the Coulomb field of an electron density inhomogeneity, can lead to attraction of the electron to an inhomogeneity of the charge density and growth of the latter.

For sub-picosecond, highly charged electron bunches [18], the NMI effect was numerically demonstrated to enable retaining of the bunch longitudinal profile during a long-distance beam line propagation through the combined magnetic field of the system, resulting in a strong THz radiation with energy extraction as high as 25% of the initial beam energy. For a single bunch, this compensation of the space-charge repulsion thanks to NMI effect was shown to be the most effective for the bunch charges of 1.0-2.5 nC. For higher bunch charges, the balance of the two opposite effects is broken, resulting in a widening of the radiation spectrum and in a remarkable reduction of the energy extraction.

Preliminary modulation of the electron density allows using of much longer electron bunches with larger charges, restricting the destructive influence of space-charge repulsion. Utilization of pre-modulated beams is supposed to enable a significant enhancement of radiation energy and narrowing of the spectra. The modulation can be obtained, for example, by means of time modulation in the illuminating laser intensity leading to a corresponding modulation of the electron current from a photocathode. A remarkable quasi-periodic self-modulation was found to be excited and developed, due to the radiation instability and thanks to NMI, also during a propagation of relatively long non-modulated electron bunches.

The simulations are carried out for the bunch parameters based on the expected parameters of the Israeli THz radiation source being constructed at Ariel University, and driven by hybrid photo-cathode RF-gun with the electron energy of up to 6 MeV [11]. The 3D space-frequency approach [23] is used for numerical analysis of the radiation emission and bunch dynamics. The method is based on an expansion of the RF electromagnetic field in terms of eigenmodes of the medium in which the field is excited and propagates. The interaction between the RF field and the particles is described by a set of coupled self-consistent equations for wave excitation and particle dynamics, expressing the evolution of the mode amplitudes and the particle's trajectories along the interaction region. The method was realized in the numerical code WB3D and has been successfully applied to the analysis of various effects in FELs (see [24] and references therein).

NMI in undulators and the method of simulations are shortly discussed in Section 2 of the present paper. Propagation and emission, in regime of NMI stabilization, of a single short electron bunch or of a relatively long beam pulse with or without harmonic pre-modulation are considered in Sections 3 and 4, respectively. A possibility of injection of a dense electron bunch into a strong magnetic field is demonstrated in Section 5.

2. Negative Mass Instability in undulators and the method of simulations

The undulator NMI can be realized in a combined undulator and uniform strong (over-resonance) longitudinal magnetic field [15-17]. It is well-known that electrons can move in such a field along the helical stationary trajectories with the normalized transverse velocity

$$\beta_\perp = K/\gamma|\Delta| \tag{1}$$

where $K = e\, B_u\, \lambda_u \,/\, 2\pi\, m_e c^2$ is the undulator parameter for the helical magnetic field with the amplitude B_u, period λ_u, and undulator frequency $\Omega_u = 2\pi c\, \beta_\parallel \,/\lambda_u$, $\Omega_c = eB_0/m_e c\gamma$ is the cyclotron frequency, and $\Delta = 1 - \Omega_c/\Omega_u$ is the relative mismatch of the resonance between free cyclotron and forced undulator particle oscillations. For the helical stationary trajectory, the derivative of the electron longitudinal velocity on the particle energy $E = m_e c^2 \gamma$ can be given (within the ultra-relativistic approach) by

$$\frac{\partial \beta_{\parallel}}{\partial \gamma} = \frac{1}{\gamma^3}\left(1 + \frac{K^2}{\Delta^3}\right). \quad (2)$$

The undulator NMI develops if this value is negative. The corresponding decrease or increase in the electron longitudinal velocity with an increase or decrease in the electron energy due to additional particle acceleration in the Coulomb field of an electron density inhomogeneity, can than lead to attraction of the electron to an inhomogeneity of the charge density and growth of the latter.

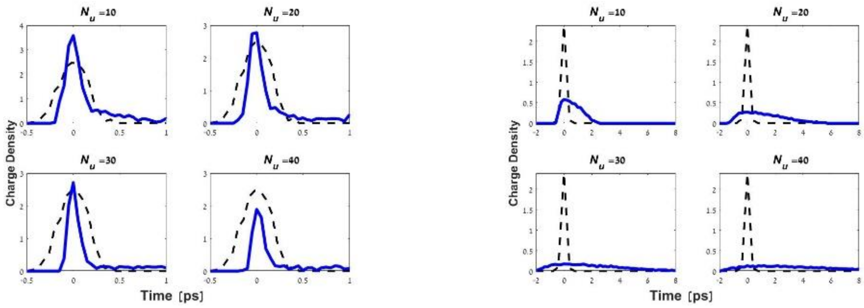
The following simulations of THz NMI free-electron radiation source were carried out in the framework of 3D space-frequency approach given in more details in [23]. The method is based on an expansion of the high-frequency electromagnetic field in terms of transverse eigen modes of the waveguide in which the field is excited and propagates. The interaction between the electromagnetic field and the gain medium is fully described by a set of coupled equations, expressing the evolution of mode amplitudes along the interaction region. The drive electron beam is introduced by a set of "macro-particles" with distribution corresponding to the density and the velocity electron distributions in the beam bunch. Trajectory of each "macro-particle" under influence of the radiation field and of the rest of "macro-particles" (what simulates the space-charge effects) is found from the motion equations, solved self-consistently along with solution for the field excitation. The approach was realized in the numerical code WB3D [23] and is used in the present work for analysis of THz coherent spontaneous undulator radiation produced in a metal waveguide by short and very dense electron bunches moving in a combined undulator field. Parameters of the drive electron bunches are close to the expected parameters of the Israeli THz source (ITS) developed at Ariel University using a hybrid laser-driven photo-injector [11] (Table I). Parameters of the first stage of the experiments planned at the ITS [11] were changed in the present research to study capabilities of the NMI: the planar undulator and the rectangular waveguide of the original project were replaced with a helical undulator combined with a strong uniform guiding field, and by a circular cylindrical waveguide, respectively. Operating waveguides with large (over-sized) cross-sections were chosen in the both configurations for the maximal increase of the radiation frequency and for a simplicity of a long-distance bunch transport. In addition to the configuration replacements mentioned above, short electron bunches with the charges of up to a few nano-Coulomb (what is much higher than it is supposed in the first-stage ITS design) are considered in the simulations.

3. NMI source driven by a single short beam bunch

When the guiding magnetic field is zero, the synchronism frequency for 6 MeV electron beam moving inside the chosen over-sized waveguide is about 3 THz. At NMI regime taking place with the "positive" direction of the "near resonance" guiding magnetic field ($\Omega_c > 0$), the longitudinal particle velocity and the Doppler-frequency shift are essentially smaller due to much stronger transverse oscillations of the electrons. Focusing magnetic field of B_u = 75 kG is supposed in the present work, and, correspondingly, the synchronism radiation frequency drops down to $f_s \approx 1.5$ THz. Very short drive electron bunches with the initial time duration of T_b = 0.3 ps ($f_s \cdot T_b \approx 0.45$) are considered here to enable a coherent spontaneous emission at this frequency.

Table I: Basic parameters of the simulated THz source.

Accelerator	
Type	Photo-injector RF-linac
Electron kinetic energy, E_k	6 MeV ($\gamma_0 \approx 13$)
Transverse bunch radius, R_b (rms)	0.5 mm
Waveguide	
Type	Circular cylindrical, metal
Radius	5 mm
Dominant modes	TE_{11}, TE_{12}, TE_{13}, TM_{11}, TM_{12}
Helical undulator	
Period, λ_u	25 mm
Field amplitude, B_u	2 kG
Undulator constant, K	0.47
Uniform guiding field, B_0	75 kG



(*a*) (*b*)

Fig.1. Charge density for 1.5 nC bunch at the undulator entrance (black line at each picture) and after each 10 periods along the beam line: the NMI (a) and the "positive mass" (b) regimes.

The characteristic duration of an initially short electron bunch with a relatively low or moderate charge, q_b = 0.5-1.5 nC, is conserved or even turns to be a bit shorter during the bunch motion in the undulator thanks to the NMI (see Fig. 1a for the example of 1.5nC bunch). The destructive space-charge force grows with the charge, breaking the balance between the repulsive and attractive forces for charges higher than 2.5 nC. The main peak of the charge density is reduced and turns to be wider along the beam trajectory, while a considerable part of the charges leaves the central peak and forms a widely-extended "tail" after it. Corresponding radiation spectrum for 1.5 nC bunch is demonstrated at Fig. 2. The spectrum includes the low-frequency and the high-frequency branches of the excited TE_{11}, TE_{12}, TE_{13}, TM_{11}, and TM_{12} waveguide modes, with dominant TE_{13} and TE_{12} modes at the high-frequency part of the spectra. For the "positive mass" regime taking place if the oppositely directed focusing magnetic field of the same value is used, the bunch quickly expands, its duration grows monotonically (Fig. 1b) and the radiation is negligibly weak.

Due to the NMI, the high-frequency radiation emission grows remarkably at the beam charges less than 3.5 nC, when the radiation saturates slowly during the bunch propagation after approximately 60 undulator periods (Fig. 3a), reaching as strong energy as 2.4 mJ for 1.5 nC charge. For higher beam charges, the energy flux remains almost constant due to fast bunch spreading. The energy extraction efficiency is evaluated as the ratio of the radiation flux and the initial bunch energy $q_b E_k$, and peaks at charges 1.0-2.5 nC when more than 25% of the initial beam energy is converted into radiation (Fig. 3b).

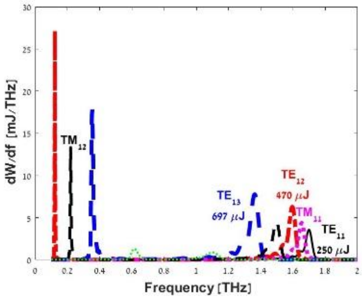
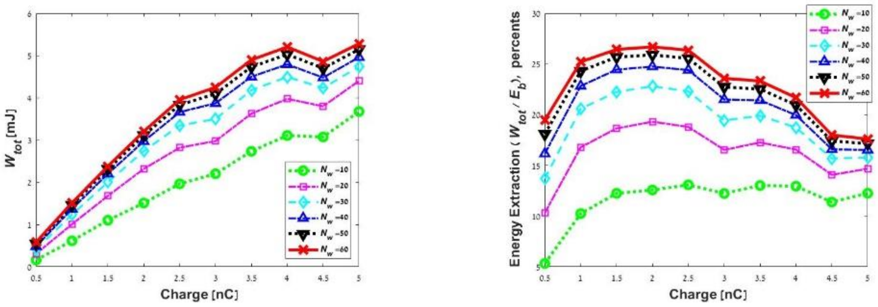


Fig.2. Spectral radiation density dW/df for charge of 1.5 nC, obtained after 40 undulator periods.



(a) (b)

Fig.3. Charge dependence of radiation energy flux (a) and energy extraction (b) gained at the high-frequency spectrum branch.

4. Evolution and radiation of long pre-modulated and non-modulated bunches

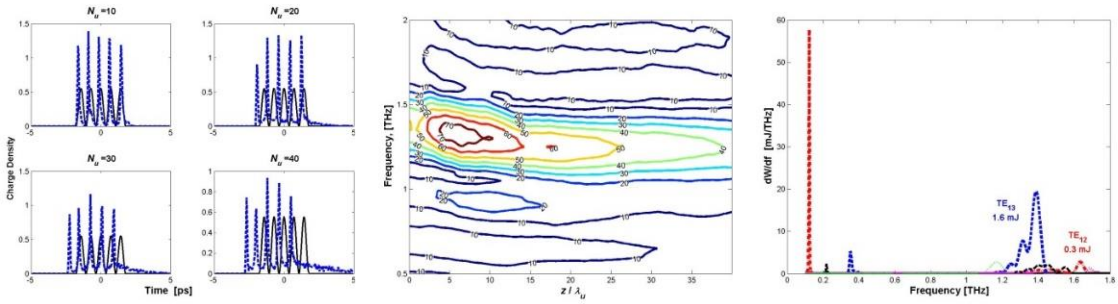
A train of short bunches like considered above can be introduced as a modulated bunch with the longitudinal charge density $\rho = \rho_0 \left[1 + m\cos\left(2\pi f_m t\right)\right]$, where f_m is the modulation frequency and m is the modulation depth. For relatively thin bunches of the radius $R_b << \gamma d$, where d is the spatial period of the modulation, mutual interaction of the nearest particles is basically important. In the NMI regime, the particles will be attracted to the nearest maxima making the latter become sharper up to saturation of the NMI. From other side, interaction between maxima is relatively weak. As a result, train of sharp, long-living density peaks is formed in NMI regime, enabling enhancement of radiation energy and more selective excitation of waveguide modes.

A deterministic distribution of macro-particles corresponding to a periodic initial density modulation with an integer number of periods $N_\mathrm{m} = f_m T_b$ at the modulation frequency f_m = 1.4 THz close to the maxima of the single-charge radiation spectrum is used in the simulations. $10^3 N_\mathrm{m}$ macro-particles are considered in the simulations in order to provide a correct description of space-charge effects in dense bunches.

For 2.5 nC beam pulse of N_m = 5 modulation periods at initially 100% modulation (m=1) (Fig. 4), train of sharp bunches is formed at the very first undulator periods of the beam propagation and holds its shape down the beamline. High coherency of the beam at the radiation frequencies near 1.4 THz can also be demonstrated by the bunching factor $\langle e^{i\,2\pi f\, t_l(z)} \rangle$ shown at Fig. 4b, here $t_l(z)$ is the time of arrival of a macro-particle l at the point z. After 40 undulator periods, the radiation energy at the strongly dominant TE_{13} mode is as high as 1.6 mJ that corresponds to a high radiation efficiency of 11% (the total radiation energy and efficiency are in this case 2.9 mJ and 19%, respectively).

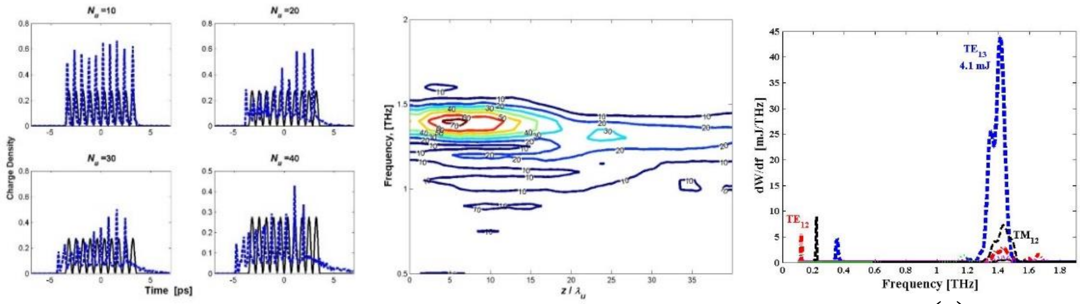
An increase in the number of the modulation periods N_m while retaining a charge per period (0.5nC/period), leads to a better mode selectivity and a higher radiation extraction efficiency (figs. 5 and 6). After 40 undulator periods, the radiation energy at the dominant TE_{13} mode is 4.1 mJ and 6.2 mJ for 10 and 15 modulation periods, respectively, that corresponds to efficiency of 14%.

Radiation extraction efficiency and mode selectivity remain comparatively high even at a significantly lower initial modulation, as demonstrated at Fig. 7 for the modulation index m=0.5 (10 modulation periods). To avoid the effects of the sharp edges of the electron density, the density distribution was sufficiently smoothed at the edges of the electron pulse. The radiation energy and efficiency at the TE_{13} mode are 2.4 mJ and 8%, respectively (the total radiation energy and efficiency are 3.8 mJ and 13%).



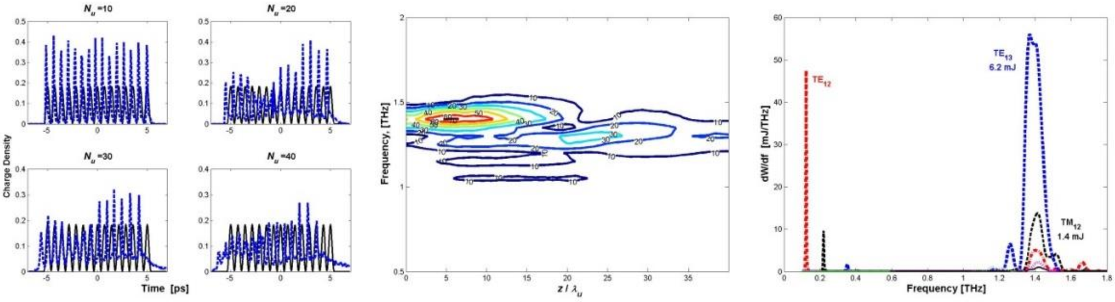
(a) (b) (c)

Fig. 4. Bunch with charge 2.5 nC and 5 periods of 100% modulation at the frequency 1.4 THz in NMI regime: (a) charge density after each 10 undulator periods, (b) map of bunching factor at high frequencies in the plane f-z, and (c) radiation spectrum after 40 undulator periods.



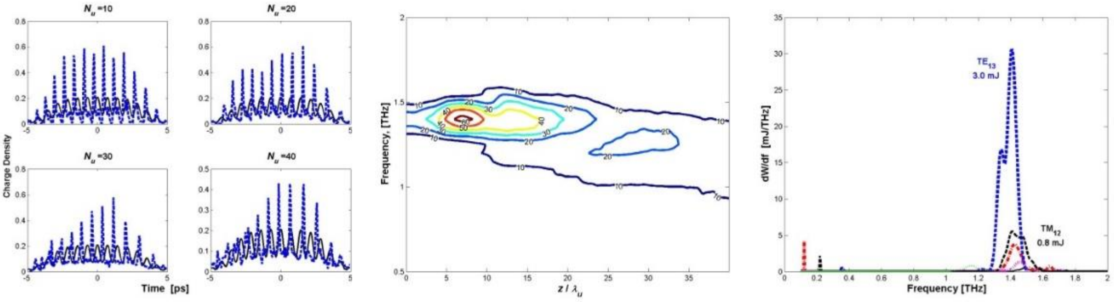
(a) (b) (c)

Fig. 5. Bunch with charge 5 nC and 10 periods of 100% modulation at the frequency 1.4 THz in NMI regime: (a) charge density after 10 to 40 undulator periods, (b) map of bunching factor at high frequencies in the plane f-z, and (c) radiation spectrum after 40 undulator periods.



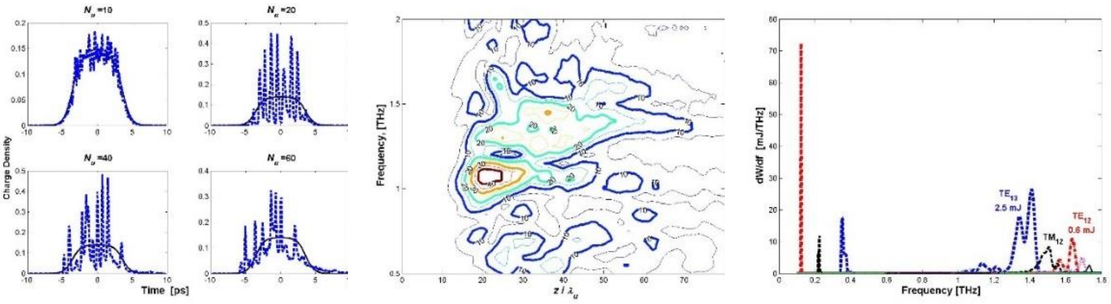
(a) (b) (c)

Fig. 6. Bunch with charge 7.5 nC and 15 periods of 100% modulation at the frequency 1.4 THz in NMI regime: (a) charge density after 10 to 40 undulator periods, (b) map of bunching factor at high frequencies in the plane *f*-*z*, and (c) radiation spectrum after 40 undulator periods.



(a) (b) (c)

Fig. 7. Bunch with charge 5 nC and 10 periods of 50% modulation at the frequency 1.4 THz and smoothed edges in NMI regime: (a) charge density after 10 to 40 undulator periods, (b) map of bunching factor at high frequencies in the plane *f*-*z*, and (c) radiation spectrum after 40 undulator periods.



(a) (b) (c)

Fig. 8. Non-modulated bunch with charge 5 nC, duration 10.6 ps, and smoothed edges in NMI regime: (a) charge density after 10 to 60 undulator periods, (b) map of bunching factor in the plane *f*-*z*, and (c) radiation spectrum after 60 undulator periods.

As is well known, the radiation is possible and can be fairly efficient even in the case of the complete absence of preliminary modulation, m=0, in a relatively long (extended) bunches due to efficient self-modulation and bunching of particles in the combination wave formed by the undulator field and the field of radiated waves. This version of super-radiance is successfully used for production of very powerful microwave generation from nanosecond electron beams (see, e.g., [25-27] and the literature cited therein). Such a mechanism of super-radiance may be also promising for sources of undulator terahertz radiation utilizing picosecond electron bunches from photo injectors.

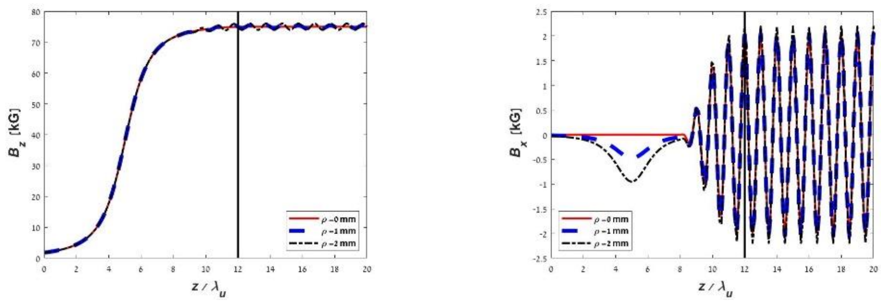
Our simulations show that a relatively long non-modulated bunch with smoothed edges is efficiently self-modulated due to the development of radiative instability with NMI-stabilization and sharping of the density distribution (Fig. 8). Almost the same radiation energy and efficiency at a good quality of the radiation spectrum are obtained after 60 undulator periods (20 additional undulator periods were added to enable an effective emission after the self-modulation process) as in the case of 50% modulation (Fig. 7).

5. Injection of a dense electron bunch into a strong magnetic field

To transport electron bunches in accelerators, a much weaker focusing magnetic field is usually used than required for NMI regime. Correspondingly, a drive electron bunch should be properly injected into the region of the combined strong focusing and undulator fields after the acceleration in RF-linac. For this reason, we consider an injection section followed by NMI interaction region as described above. The injection section includes an entrance into a long solenoid of 4 cm radius and a helical undulator with an adiabatically increasing field strength; the undulator includes 4 periods of 2.5 cm. Total length of the injection section is 30 cm. The longitudinal B_z and the transverse B_x magnetic fields in the front-end side of the system are shown at the Fig. 9a, b, where the solenoid and undulator begin at z = 12.5cm and z = 20 cm, respectively.

Results for 1 nC bunch of 0.6 ps initial time duration are presented here. To enable the particles' injection along the force lines of the magnetic field and to prevent excitation of strong cyclotron oscillations of the beam, a convergent beam with the transverse velocities of the particles being proportional to the particle distance to the axis $\beta_\perp \sim \varrho$ is considered at the entrance. Such a distribution of the transverse electron velocities corresponds to propagation of the electron beam through a focusing electric lens. Energy chirp of 0.6 MeV is additionally added to limit the bunch spreading in the injection section.

Trajectory of the bunch is demonstrated at Fig. 10a, where red lines show the force lines of the field. The adiabatic bunch injection enables formation of a sharp beam peak, existing for 40 undulator periods at least (Fig. 10b) and emitting more than 0.3 mJ radiation at the high-frequency spectrum branch (5.4% of the energy extraction; 137μJ and 2.3%, respectively, for the TE_{13} mode only).



(a) (b)

Fig. 9. The longitudinal B_z (a) and the transverse B_x magnetic fields in the front-end of the system. End of the injection section is shown by vertical black line.

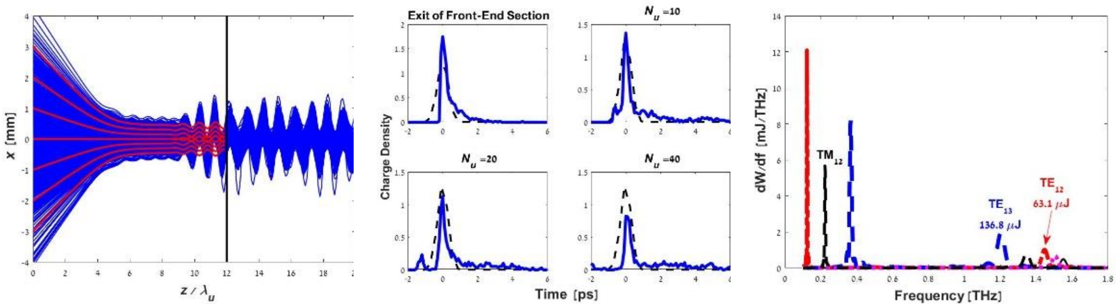


Fig. 10. Injection of a 1 nC bunch into the NMI region: trajectories of the particles (a), the bunch charge density (b), and the output spectrum (c).

6. Conclusion

A promising possibility for intensive coherent spontaneous undulator radiation at THz frequency range emitted by short (sub-picosecond) dense electron bunches is demonstrated in simulations for the conditions of NMI stabilization of their longitudinal sizes. It is believed that such bunches can be obtained from a hybrid RF-linac by means of a bunch energy chirping followed by its ballistic compression before the utilization. The excited radiation in the chosen over-sized waveguide consists of a few dominant waveguide modes with a narrow spectrum and is characterized by an extremely high energy extraction (up to 25%).

To reduce destructive space-charge repulsion and enable using of high-charged beams, hardly achievable short dense electron bunches can be replaced by relatively long (picosecond), a few nano-Coulomb charge beam pulses. These pulses can be pre-modulated, forming a periodic train of short bunches, which emits almost a single-mode, narrow-band radiation spectrum. A remarkable quasi-periodic self-modulation is revealed to be excited and developed, due to the radiation instability and thanks to NMI, during a propagation of non-modulated picosecond electron pulses (similar to effects of super-radiance that are well known for various devices in positive-mass regimes; see, e.g., [25-27]). Although the energy extraction for long modulated or non-modulated pulses is lower than that for a single short bunch, it is still rather high (10-15%).

A possibility for injection of a dense electron bunch into a strong magnetic field required for realization of NMI conditions is preliminary demonstrated. Even if this challenging issue should be additionally studied to improve the beam transport through the injection section, a practical realization of compact and powerful sources for intense and narrow-band THz radiation seems to be possible.

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7. References

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