Retardation Effects in Electromagnetism and Gravitation

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Abstract

We study the effect of retardation in both electromagnetic and gravitational systems and show its implication to both large systems and fast changing systems in which those effects are considerable.

1 Introduction

Among the major achievements of Sir Isaac Newton is the formulation of Newton's third law stating that any action is countered by a reaction of equal magnitude but opposite direction [1, 2]. The total force in a system not affected by external forces is thus zero. This law has numerous experimental verifications and seems to be one of the corner stones of physics. However, by the middle of the nineteenth century Maxwell has formulated the laws of electromagnetism in his famous four partial differential equations [3, 4, 5]. One of the consequences of these equations is that an electromagnetic signal cannot travel at speeds exceeding that of light. This was later used by Albert Einstein [6, 4, 5] (among other things) to formulate his special theory of relativity which postulates that the speed of light is the maximal allowed velocity in nature. According to the principles of relativity no signal (even if not electromagnetic) can propagate at superluminal velocities. Hence an action and its reaction cannot be generated at the same time because of the relativity of simultaneity. Thus the total force cannot be null at a given time. In consequence, by not holding rigorously the simultaneity of action and reaction Newton's third law cannot hold in exact form but only as an approximation. Moreover, the total force within a system that is not acted upon by an external force would not be rigorously null as will be shown in this paper for electromagnetic systems.

The general theory of relativity (GR) is verified by many observations. Nevertheless, some observations seems not to fit GR and observed matter. As soon as 1933 Fritz Zwicky realized that the velocities of the Galaxies within the Comma Cluster are way larger than those predicted by the virial theorem in Newtonian theory [8]. He remarked that the amount of matter needed to account for the velocities could be 400 times that of the visible matter. Which led to postulating an unseen form of matter permeating the cluster. Volders in 1959 remarked that stars in the periphery of the neighbor spiral

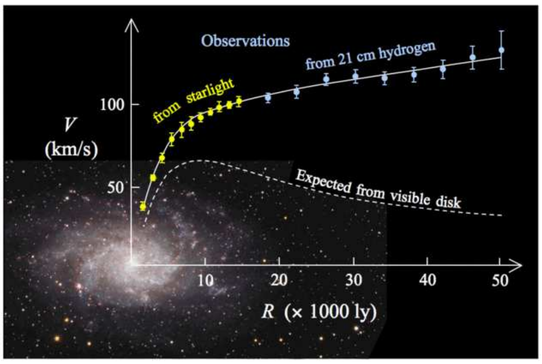

Figure 1: M33 rotation curve [12]

galaxy M33 do not move as expected [9]. The virial theorem in Newtonian Gravity predicts that $MG/r \sim Mv^2$, that is to say, the rotation curve should increase and at some point bend down and the velocity should drop off as $1/\sqrt{r}$. In the seventies Rubin and Ford [10, 11] showed for a very large sample of spiral galaxies that this behavior is a general feature: velocities at the periphery of the galaxies do not bend down, attain a plateau at some velocity for each galaxy. In figure 1 we see a rotation curves for the M33 galaxy describing this situation. In what follows we will show that such effects can be deduced from GR if retardation effects are not neglected.

2 Electromagnetism

Let us [19] consider the general time dependent case. Maxwell's equations dictate that in this case one can not have a magnetic field without an electric field and vice versa. Therefore we will consider both the electric and magnetic parts of the Lorenz force $\vec{F}_{21}$. Let us suppose that the electric field $\vec{E}$ and magnetic field $\vec{B}$ are created by current loop 1 and acts upon current loop 2. Since a conductive loop that carries a neutral charge will contain both ions and free electrons (in equal amounts) we will have:

$$\vec{F}_{21} = \int d^3x_2 \rho_{i2}(\vec{E} + \vec{v}_{i2} \times \vec{B}) + \int d^3x_2 \rho_{e2}(\vec{E} + \vec{v}_{e2} \times \vec{B}). \tag{1}$$

In the above we integrate over the entire volume of current loop 2. ρ_{i2} and ρ_{e2} are the ion charge density and electron charge density respectively, $\vec{v}_{i2}$ and $\vec{v}_{e2}$ are the ion velocity field and electron velocity field respectively. Since the amounts of ions and free electrons are equal we will assume:

$$\rho_{i2} = -\rho_{e2}. \tag{2}$$

Thus the electric terms in the above force equation cancel and we are left with:

$$\vec{F}_{21} = \int d^3x_2 \rho_{i2} \vec{v}_{i2} \times \vec{B} + \int d^3x_2 \rho_{e2} \vec{v}_{e2} \times \vec{B}. \tag{3}$$

In the laboratory frame the ions being at rest we have: $\vec{v}_{i2} = 0$. Thus we arrive at the expression:

$$\vec{F}_{21} = \int d^3x_2 \rho_{e2} \vec{v}_{e2} \times \vec{B} \tag{4}$$

Introducing the current density: $\vec{J}_2 = \rho_{e2}\vec{v}_{e2}$ we obtain the expression:

$$\vec{F}_{21} = \int d^3x_2 \vec{J}_2 \times \vec{B}. \tag{5}$$

Now, let us consider the coil that generates the magnetic field. The magnetic field can be written as follows in terms of its vector potential [4]:

$$\vec{B} = \vec{\nabla} \times \vec{A}. \tag{6}$$

If the field is generated by a current density $\vec{J}_1$ in coil 1 we can solve for the vector potential and obtain the result [4]:

$$\vec{A}(\vec{x}_2) = \frac{\mu_0}{4\pi} \int d^3x_1 \frac{\vec{J}_1(\vec{x}_1, t_{ret})}{R}, \qquad \vec{R} \equiv \vec{x}_{12}, \qquad t_{ret} \equiv t - \frac{R}{c}. \tag{7}$$

In the above t is time and c is the speed of light in vacuum. Combining equation (7) with equation (6) we arrive at the result:

$$\vec{B}(\vec{x}_2) = \vec{\nabla}_{\vec{x}_2} \times \vec{A}(\vec{x}_2) = \frac{\mu_0}{4\pi} \int d^3x_1 \vec{\nabla}_{\vec{x}_2} \times \left. \frac{\vec{J}_1(\vec{x}_1, t_{ret})}{R} \right). \tag{8}$$

However, notice that[1]:

$$\vec{\nabla}_{\vec{x}_2} \times \left. \frac{\vec{J}_1(\vec{x}_1, t_{ret})}{R} \right) = \vec{\nabla}_{\vec{x}_2} R \times \partial_R \left. \frac{\vec{J}_1(\vec{x}_1, t_{ret})}{R} \right). \tag{9}$$

Since:

$$\vec{\nabla}_{\vec{x}_2} R = -\frac{\vec{R}}{R} \tag{10}$$

And:

$$\partial_R \left. \frac{\vec{J}_1(\vec{x}_1, t_{ret})}{R} \right) = -\frac{\vec{J}_1(\vec{x}_1, t_{ret})}{R^2} - \frac{\partial_t \vec{J}_1(\vec{x}_1, t_{ret})}{Rc}. \tag{11}$$

Hence:

$$\vec{\nabla}_{\vec{x}_2} \times \left. \frac{\vec{J}_1(\vec{x}_1, t_{ret})}{R} \right) = \frac{\vec{R}}{R^3} \times \left(\vec{J}_1(\vec{x}_1, t_{ret}) + \left(\frac{R}{c}\right) \partial_t \vec{J}_1(\vec{x}_1, t_{ret}) \right). \tag{12}$$

Inserting equation (12) into equation (8) we arrive at Jefimenko's equations [4]:

$$\vec{B}(\vec{x}_2) = \frac{\mu_0}{4\pi} \int d^3x_1 \frac{\vec{R}}{R^3} \times \left(\vec{J}_1(\vec{x}_1, t_{ret}) + \left(\frac{R}{c}\right) \partial_t \vec{J}_1(\vec{x}_1, t_{ret}) \right). \tag{13}$$

Although this derivation is standard it is repeated here for completeness. The current density in a thin-conductor loop can be expressed in terms of the loop's current as follows:

$$\int d^3x_1 g(\vec{x}_1) \vec{J}_1(\vec{x}_1, t) = \int dl_1 g(\vec{x}_1) \int dA_1 \vec{J}_1(\vec{x}_1, t) = \int d\vec{l}_1 g(\vec{x}_1) I_1(t). \tag{14}$$

[1]We use the notation $\partial_y \equiv \frac{\partial}{\partial y}$.

In the above dA_1 is a cross section area element of the loop and $g(\vec{x}_1)$ is an arbitrary function. In terms of the result given in equation (14) one may write equation (13) as:

$$\vec{B}(\vec{x}_2,t) = \frac{\mu_0}{4\pi}\oint \frac{\vec{R}}{R^3} \times d\vec{l_1}\left(I_1(t_{ret}) + \left(\frac{R}{c}\right)\partial_t I_1(t_{ret})\right). \tag{15}$$

Inserting equation (15) into equation (5) we arrive at the result:

$$\vec{F}_{21} = \frac{\mu_0}{4\pi}\int d^3x_2 \vec{J}_2 \times \oint \frac{\vec{R}}{R^3} \times d\vec{l_1}\left(I_1(t_{ret}) + \left(\frac{R}{c}\right)\partial_t I_1(t_{ret})\right). \tag{16}$$

Assuming that current loop 2 has also a small cross section area and using the same argument as in equation (14) we arrive at the result:

$$\vec{F}_{21} = \frac{\mu_0}{4\pi}I_2(t)\oint d\vec{l_2} \times \oint \frac{\vec{R}}{R^3} \times d\vec{l_1}\left(I_1(t_{ret}) + \left(\frac{R}{c}\right)\partial_t I_1(t_{ret})\right). \tag{17}$$

2.1 The Case of a Finite τ

Consider the current $I(t_{ret}) = I(t-\frac{R}{c})$, if $\frac{R}{c}$ is small but not zero one can write a Taylor series expansion around t in the form:

$$I(t_{ret}) = I(t-\frac{R}{c}) = \sum_{n=0}^{\infty}\frac{I^{(n)}(t)}{n!}(-\frac{R}{c})^n. \tag{18}$$

In the above $I^{(n)}(t)$ is the derivative of order n of $I(t)$. Inserting equation (18) into equation (7) and taking into account equation (14) we obtain:

$$\vec{A}(\vec{x}_2) = \frac{\mu_0}{4\pi}\oint d\vec{l_1}\frac{I_1(t_{ret})}{R} = \frac{\mu_0}{4\pi}\sum_{n=0}^{\infty}\frac{I_1^{(n)}(t)}{n!}\oint d\vec{l_1}\frac{1}{R}(-\frac{R}{c})^n \tag{19}$$

Denoting $g_n(R) = \frac{1}{R}(-\frac{R}{c})^n$ and inserting equation (19) into equation (6) we obtain:

$$\vec{B}(\vec{x}_2) = \frac{\mu_0}{4\pi}\sum_{n=0}^{\infty}\frac{I_1^{(n)}(t)}{n!}\oint \vec{\nabla}_{\vec{x}_2} \times (d\vec{l_1}g_n(R)) \tag{20}$$

Now writing equation (5) in terms of equation (20) and taking into account equation (14) we have:

$$\vec{F}_{21} = \oint d\vec{l_2}I_2(t) \times \vec{B} = \frac{\mu_0}{4\pi}I_2(t)\sum_{n=0}^{\infty}\frac{I_1^{(n)}(t)}{n!}\oint\oint d\vec{l_2} \times (\vec{\nabla}_{\vec{x}_2}g_n(R) \times d\vec{l_1}). \tag{21}$$

Using a well known vector identity:

$$d\vec{l_2} \times (\nabla_{\vec{x}_2}g_n(R) \times d\vec{l_1}) = \vec{\nabla}_{\vec{x}_2}g_n(R)(d\vec{l_2}\cdot d\vec{l_1}) - d\vec{l_1}(d\vec{l_2}\cdot\vec{\nabla}_{\vec{x}_2}g_n(R)) \tag{22}$$

We can write:

$$\oint \oint d\vec{l_2} \times (\vec{\nabla}_{\vec{x}_2} g_n(R) \times d\vec{l_1}) = \oint \oint \vec{\nabla}_{\vec{x}_2} g_n(R)(d\vec{l_2} \cdot d\vec{l_1}) - \oint d\vec{l_1} \oint d\vec{l_2} \cdot \vec{\nabla}_{\vec{x}_2} g_n(R) \quad (23)$$

but since $\oint d\vec{l_2} \cdot \vec{\nabla}_{\vec{x}_2} g_n(R) = 0$ equation (21) can be written as:

$$\vec{F}_{21} = \frac{\mu_0}{4\pi} I_2(t) \sum_{n=0}^{\infty} \frac{I_1^{(n)}(t)}{n!} \oint \oint \vec{\nabla}_{\vec{x}_2} g_n(R)(d\vec{l_2} \cdot d\vec{l_1}). \quad (24)$$

The force takes now the form:

$$\vec{F}_{21} = \frac{\mu_0}{4\pi} I_2(t) \sum_{n=0}^{\infty} \frac{I_1^{(n)}(t)}{n!} (-\frac{1}{c})^n (1-n) \oint \oint R^{n-3} \vec{R}(d\vec{l_2} \cdot d\vec{l_1}). \quad (25)$$

We note that there is no first order contribution to the force. Hence the next contribution to the force after the quasi-static term is second order. Let us define the dimensionless geometrical factor $\vec{K}_{21n}$ as:

$$\vec{K}_{21n} = \frac{1}{h^n} \oint \oint R^{n-3} \vec{R}(d\vec{l_2} \cdot d\vec{l_1}) = -\vec{K}_{12n}. \quad (26)$$

in the above h is some characteristic distance between the coils. In terms of $\vec{K}_{21n}$ we can write equation (25) as:

$$\vec{F}_{21} = \frac{\mu_0}{4\pi} I_2(t) \sum_{n=0}^{\infty} \frac{I_1^{(n)}(t)}{n!} (-\frac{h}{c})^n (1-n) \vec{K}_{21n}. \quad (27)$$

The force due to coil 2 that acts on coil 1 is:

$$\vec{F}_{12} = \frac{\mu_0}{4\pi} I_1(t) \sum_{n=0}^{\infty} \frac{I_2^{(n)}(t)}{n!} (-\frac{h}{c})^n (1-n) \vec{K}_{12n}. \quad (28)$$

The total force on the system is thus:

$$\vec{F}_T = \vec{F}_{12} + \vec{F}_{21} = \frac{\mu_0}{4\pi} \sum_{n=0}^{\infty} \frac{(1-n)}{n!} (-\frac{h}{c})^n \vec{K}_{12n} \left(I_1(t) I_2^{(n)}(t) - I_2(t) I_1^{(n)}(t) \right). \quad (29)$$

We note that the quasi-static term $n = 0$ does not contribute to the sum nor does the $n = 1$ term. The fact that the retarded field "corrects" itself to first order in order to "mimic" a non retarded field was already noticed by Feynman [5]. Hence we can write:

$$\vec{F}_T = \frac{\mu_0}{4\pi} \sum_{n=2}^{\infty} \frac{(1-n)}{n!} (-\frac{h}{c})^n \vec{K}_{12n} \left(I_1(t) I_2^{(n)}(t) - I_2(t) I_1^{(n)}(t) \right). \quad (30)$$

We conclude that in general Newton's third law is not satisfied, taking the leading non-vanishing terms in the above sum we obtain:

$$\begin{aligned} \vec{F}_T &\cong -\frac{\mu_0}{8\pi} (\frac{h}{c})^2 \vec{K}_{122} \left(I_1(t) I_2^{(2)}(t) - I_2(t) I_1^{(2)}(t) \right) \\ &= -\frac{\mu_0}{8\pi} (\frac{h}{c})^2 \vec{K}_{122} I_1(t) I_2(t) \left(\frac{I_2^{(2)}(t)}{I_2(t)} - \frac{I_1^{(2)}(t)}{I_1(t)} \right). \end{aligned} \quad (31)$$

3 General Relativity

Except for the extreme cases of compact objects (black holes and neutron stars) and the very early universe (big bang) one need not consider the full non-linear Einstein equation. In most other cases of astronomical interest (galactic dynamics included) one can linearize those equations around the flat Lorentz metric $\eta_{\mu\nu}$ such that[2]:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \eta_{\mu\nu} \equiv \text{ diag } (1, -1, -1, -1), \quad |h_{\mu\nu}| \ll 1 \tag{32}$$

One than defines the quantity:

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h, \quad h = \eta^{\mu\nu}h_{\mu\nu}, \tag{33}$$

$\bar{h}_{\mu\nu} = h_{\mu\nu}$ for non diagonal terms. For diagonal terms:

$$\bar{h} = -h \Rightarrow h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}. \tag{34}$$

It can be shown ([13] page 75 exercise 37), that one can choose a gauge such that the Einstein equations are:

$$\bar{h}_{\mu\nu,\alpha}{}^{\alpha} = -\frac{16\pi G}{c^4}T_{\mu\nu}, \qquad \bar{h}_{\mu\alpha,}{}^{\alpha} = 0. \tag{35}$$

Equation (35) can always be integrated to take the form [4][3]:

$$\bar{h}_{\mu\nu}(\vec{x}, t) = -\frac{4G}{c^4}\int \frac{T_{\mu\nu}(\vec{x}', t - \frac{R}{c})}{R} d^3x',$$
$$t \equiv \frac{x^0}{c}, \quad \vec{x} \equiv x^a \quad a, b \in [1, 2, 3], \quad \vec{R} \equiv \vec{x} - \vec{x}', \quad R = |\vec{R}|. \tag{36}$$

The factor before the integral is small: $\frac{4G}{c^4} \simeq 3.3\ 10^{-44}$ hence in the above calculation one can take $T_{\mu\nu}$ which is zero order in $h_{\alpha\beta}$. Let us now calculate the affine connection in the linear approximation:

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\eta^{\alpha\beta}\left(h_{\beta\mu,\nu} + h_{\beta\nu,\mu} - h_{\mu\nu,\beta}\right). \tag{37}$$

The affine connection has only first order terms, hence for a first order approximation of $\Gamma^{\alpha}_{\mu\nu}u^{\mu}u^{\nu}$ appearing in the geodesic, $u^{\mu}u^{\nu}$ is zeroth order. In the zeroth order:

$$u^0 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad u^a = \vec{u} = \frac{\frac{\vec{v}}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad \vec{v} \equiv \frac{d\vec{x}}{dt}, \quad v = |\vec{v}|. \tag{38}$$

For non relativistic velocities:

$$u^0 \simeq 1, \qquad \vec{u} \simeq \frac{\vec{v}}{c}, \qquad u^a \ll u^0 \qquad \text{for} \quad v \ll c. \tag{39}$$

[2]Private communication with the late Professor Donald Lynden-Bell
[3]For reasons why the symmetry between space and time is broken see [14, 15]

Inserting equation (37) and equation (39) in the geodesic equation we arrive at the approximate form:

$$\frac{dv^a}{dt} \simeq -c^2\Gamma^a_{00} = -c^2\left(h^a_{0,0} - \frac{1}{2}h_{00,}{}^a\right) \tag{40}$$

Let us now look at $T_{\mu\nu} = (p+\rho c^2)u_\mu u_\nu - p\ g_{\mu\nu}$. In the current case $\rho c^2 \gg p$, combining this with equation (39) we arrive at $T_{00} = \rho c^2$ while all other components of the tensor $T_{\mu\nu}$ are significantly smaller. This implies that $\bar{h}_{00}$ is significantly larger than other components of the tensor $\bar{h}_{\mu\nu}$. Of course one should be careful and not deduce from the different magnitudes of quantities that such a difference exist between their derivatives. In fact by the gauge condition in equation (35):

$$\bar{h}_{\alpha 0,}{}^0 = -\bar{h}_{\alpha a,}{}^a \qquad \Rightarrow \bar{h}_{00,}{}^0 = -\bar{h}_{0a,}{}^a, \quad \bar{h}_{b0,}{}^0 = -\bar{h}_{ba,}{}^a. \tag{41}$$

Hence the zeroth derivative of $\bar{h}_{00}$ (contains a $\frac{1}{c}$ factor) is the same order as the spatial derivative of $\bar{h}_{0a}$ and like wise the zeroth derivative of $\bar{h}_{0a}$ (which appears implicitly in equation (40)) is the same order of the spatial derivative of $\bar{h}_{ab}$. However, it is safe to compare spatial derivatives of $\bar{h}_{00}$ and $\bar{h}_{ab}$ and conclude that the former is significantly larger than the later. Using equation (34) and taking the above consideration into account we write equation (40) as:

$$\frac{dv^a}{dt} \simeq \frac{c^2}{4}\bar{h}_{00,}{}^a \Rightarrow \frac{d\vec{v}}{dt} = -\vec{\nabla}\phi = \vec{F}, \qquad \phi \equiv \frac{c^2}{4}\bar{h}_{00} \tag{42}$$

Thus ϕ is a gravitational potential of the motion which can be calculated using equation (36):

$$\phi = \frac{c^2}{4}\bar{h}_{00} = -\frac{G}{c^2}\int \frac{T_{00}(\vec{x}', t - \frac{R}{c})}{R} d^3x' = -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x' \tag{43}$$

and $\vec{F}$ is the force per unit mass. If ρ is static we are in the realm of the Newtonian instantaneous action at a distance theory. However, it is unlikely that ρ is static as a galaxy will attract mass from the intergalactic medium.

3.1 Beyond the Newtonian Approximation

The retardation time $\frac{R}{c}$ which may be a few tens of thousands of years is short with respect to the time that the galactic density changes significantly. This means that we can write a Taylor series for the density:

$$\rho(\vec{x}', t - \frac{R}{c}) = \sum_{n=0}^{\infty} \frac{1}{n!}\rho^{(n)}(\vec{x}', t)(-\frac{R}{c})^n, \qquad \rho^{(n)} \equiv \frac{\partial^n \rho}{\partial t^n}. \tag{44}$$

Inserting equation (44) into equation (43) and keeping the first three terms we will obtain:

$$\phi = -G\int \frac{\rho(\vec{x}', t)}{R} d^3x' + \frac{G}{c}\int \rho^{(1)}(\vec{x}', t) d^3x' - \frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}', t) d^3x' \tag{45}$$

The first term will provide the Newtonian potential, the second term does not contribute, the third term will result in the lower order correction to the Newtonian theory:

$$\phi_r = -\frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}', t)d^3x' \tag{46}$$

The total force per unit mass:

$$\begin{aligned}
\vec{F} &= \vec{F}_N + \vec{F}_r \\
\vec{F}_N &= -\vec{\nabla}\phi_N = -G\int \frac{\rho(\vec{x}', t)}{R^2}\hat{R}d^3x', \qquad \hat{R} \equiv \frac{\vec{R}}{R} \\
\vec{F}_r &\equiv -\vec{\nabla}\phi_r = \frac{G}{2c^2}\int \rho^{(2)}(\vec{x}', t)\hat{R}d^3x'
\end{aligned} \tag{47}$$

While the Newtonian force $\vec{F}_N$ is always attractive the retardation force $\vec{F}_r$ can be either attractive or repulsive. Also notice that while the Newtonian force decreases as $\frac{1}{R^2}$, the retardation force is independent of distance as long as the Taylor approximation of equation (44) is valid. For short distances the Newtonian force is dominant but as the distances increase the retardation force becomes dominant. Newtonian force can be neglected for distances significantly larger than the retardation distance:

$$R \gg R_r \equiv c\Delta t \tag{48}$$

Δt is the typical duration in which the density ρ changes. Of course for $R \ll R_r$ the retardation effect can be neglected and only Newtonian forces should be considered. For large distances $r = |\vec{x}| \to \infty$ such that $\hat{R} \simeq \frac{\vec{x}}{|\vec{x}|} \equiv \hat{r}$ we obtain:

$$\vec{F}_r = \frac{G}{2c^2}\hat{r}\int \rho^{(2)}(\vec{x}', t)d^3x' = \frac{G}{2c^2}\hat{r}\ddot{M}, \qquad \ddot{M} \equiv \frac{d^2M}{dt^2}. \tag{49}$$

Now as the galaxy attracts intergalactic gas its mass increases thus $\dot{M} > 0$, however, as the intergalactic gas is depleted the rate at which the mass increases must decrease hence $\ddot{M} < 0$. Thus in the galactic case:

$$\vec{F}_r = -\frac{G}{2c^2}|\ddot{M}|\hat{r} \tag{50}$$

and the retardation force is attractive.

3.2 Dark Matter

In what circumstances can one confuse retardation with the effect of a non existent "dark matter"? Let us ignore retardation effects and suppose that radial velocities are a result of some mysterious dark matter. In this case we can write for a spherically symmetric mass distribution [16]:

$$-\frac{v_c^2}{r}\hat{r} = \vec{F}_d = -\frac{GM_d(r)}{r^2}\hat{r} \tag{51}$$

v_c is the speed of a test particle of constant radius r and $M_d(r)$ is the amount of dark matter inside the radius r. Comparing equation (51) and equation (50) we see that the "dark matter" mass can be calculated as follows:

$$M_d(r) = \frac{r^2|\ddot{M}|}{2c^2} \tag{52}$$

Now since:

$$M_d(r) = 4\pi \int_0^r r'^2 \rho_d(r') dr', \qquad \frac{dM_d(r)}{dr} = 4\pi r^2 \rho_d(r) \tag{53}$$

it follows:

$$\rho_d(r) = \frac{|\ddot{M}|}{4\pi c^2 r} \tag{54}$$

and for asymptotic radii much bigger than both the galactic radius and retardation length:

$$v_c = \sqrt{\frac{G}{2c^2}|\ddot{M}|r} \tag{55}$$

This is consistent with observational data of [12] who concluded that the "dark matter" density decreases as $r^{-1.3}$ for M33.

3.3 MOND

Another approach to explaining galactic rotation curves is the claim that either the laws of dynamics (Newton's second law) or the laws of Gravitation (GR) should be modified. This approach championed by Milgorm is denoted "MOND" (Modified Newtonian dynamics) [17]. In one version of this approach Newton's law of gravity is modified:

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r} \tag{56}$$

In the above μ is the interpolation function that should be 1 for $a_0 \ll a$. Let us assume:

$$\mu(\frac{a}{a_0}) = \frac{1}{1+(\frac{a_0}{a})^2} \tag{57}$$

If $a_0 \gg a$, $\mu \simeq (\frac{a}{a_0})^2$. A test particle revolving in a constant radius will have centrifugal acceleration $a = \frac{v^2}{r}$ and thus:

$$\vec{F}_M = -\frac{GMa_0^2}{v^4}\hat{r} \tag{58}$$

For v constant at a far away distance this expression is similar to the retardation force and thus:

$$|\ddot{M}| = \frac{2Ma_0^2c^2}{v^4}. \tag{59}$$

Milgrom found $a_0 = 1.2\ 10^{-10} ms^{-2}$ to be most fitting to the data. The mass of the M33 galaxy is $9.95\ 10^{40} kg$ and the velocity far away[4] from the galaxy is $179,000\ ms^{-1}$. We

[4] This is only a crude estimation, of course the retardation force does not allow strictly constant circular velocity profiles

thus obtain $\ddot{M} \simeq 2.51\ 10^{17} kgs^{-2}$ and a ratio $\frac{|\ddot{M}|}{M} \simeq 2.52\ 10^{-24}\ s^{-2}$. This amounts to a typical accumulation acceleration time scale of $20,000$ years and retardation distance of $20,000$ light years which seems reasonable according to figure 1.

4 Conclusion

We show that retardation means that Newton third law does not hold exactly and the deviation from this law will be manifested in electromagnetic systems of large current second derivatives of [19, 20, 21], that is fast changing currets.

We show that "dark matter" and "MOND" effects are explained in the framework of standard GR as effects due to retardation without assuming any exotic matter or modifications of the theory of gravity.

What will happen if the mass out side the galaxy is not yet depleted? In this case $\ddot{M} = 0$ and retardation force should vanish. This was indeed reported recently [18] for the galaxy NGC1052-DF2.

References

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