Inertial confinement as a model of particles interacting at high energies and a correction to the Debye length

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Abstract

We discuss an application of ideas of inertial confinement from plasma physics for the gas of charged particles interacting at high energy. We formulate this problem as a problem of equilibrium state for the "internal" (slow) particles in some additional potential field created by "external" (fast) particles. In this formulation we find an equation of state for the "internal" particles. We also consider a shock wave description of the process discussing a self-similar solution ofthe Euler's equations for the fields of interest and, finally, we solve Boltzmann equation for self-consistent field (Vlasov's equations) in linear approximation determining an electrical properties of collisionless plasma.

1 Introduction

A collision of heavy nuclei at LHC offers new possibilities to explore a new state of matter, the Quark Gluon Plasma state, which is a dense state of strongly interacting quarks and gluons. Initial stage of scattering of two nuclei is an initial condition for further evolution of the bulk of quarks and gluons in the form of the ideal fluid, see [1]. From this point of view it is interesting to understand a creation of the very dense areas of the mater at initial stages of scattering.

The process of nuclei-nuclei scattering at high energy is a highly non-equilibrium process due to the presence of the partons with different distribution over the particles velocities (rapidities), see [2] for example. In order to describe processes of interactions between these partons we use the following approach, see [3]. Part of the bulk of the particles interacting firstly at the very initial stages of collision forms an area of higher density due to the first initial nuclei fronts interactions. We will call these particles as internal ("slow") particles. Namely, these particles loose their velocities during the interactions and heat the interaction area providing some spots of high partons density. There are also coming later secondary partons which we will call an external or "fast" ones.

The main separation between these "slow" and "fast" particles comes from the value of their relative rapidity - the "fast"particles have a large rapidity in relation to the bulk of

”slow” particles. Following by [3], the interactions of these internal particles with external ones we describe as an equilibrium process for the internal particles in some additional potential field. Namely, the influence of the ”fast” particles we reduce to some effective potential field included in the description of the matter state. The advantage of this approach is clear, in this way we reduce the non-equilibrium process to the equilibrium one with the possibility to apply a wide variety of approaches knowing for equilibrium description of the matter states.

The whole process description, therefore, is similar to the effect of *inertial confinement* in plasma physics, see [4], where for the creation of the requested density of the matter needed for the thermonuclear fusion a radiation pressure on an external area of a target is used, see Fig.1.

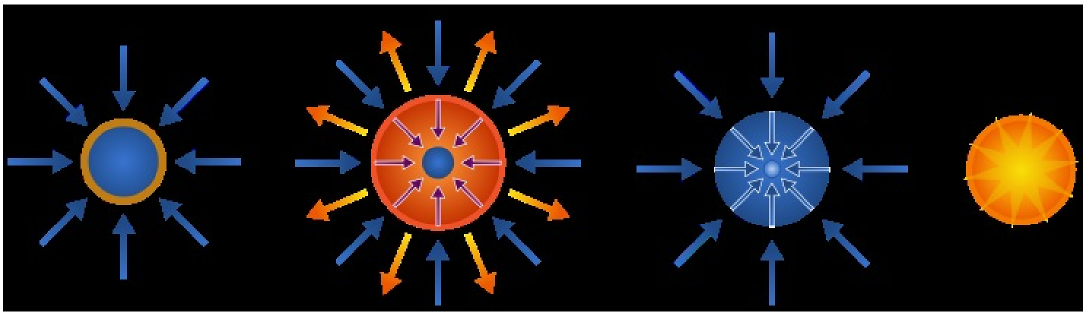

Fig.1. The principle of inertial confinement: an axternal radiation explodes the outer shel of the target, which rapidly tears from it outwards. Due to the momentum conservaation, the rest part of the target is stressed and shrinked increasing the pressure needed to the ignition of thermonuclear processes.

In our paper, in order to understand a qualitative behavior of the internal particles, we consider an oversimplified model of interacting particle. We assume that the gas of our particles is the gas of charged disks with some additional interparticle interactions which depend on *rapidity* variable. The choise of this variable as the main variable which separates different scales in the process is dictated by the QCD Pomeron approach in the high energy physics, see [5].

Thus, in Section 1 of the paper, we consider the interactions between our ”toy” particles with very general pairwise potential in two dimensional space. The additional external field is chosen to be depended on the rapidity variable and it is the only characteristic of the external particles in the model. For our purposes the form of the potential is unimportant and we chose some simple one. The following subsections of this section are dedicated to the Van der Vaals equation of state of the gas.

In Section 2, we consider Boltzmann's equation in *Vlasov's formulation* for collisonless plasma in three dimensions. In subsections of this section we find an equilibrium state and its deviation for the distribution function of the particles. Last section is dedicated to the discussion of the obtained results and the conclusion of the paper is given.

2 Boltzmann equation for self-consistent field

In this section we consider a kinetic approach to our problem. Namely, we consider a gas of charged particles of the same kind interacting in 3 dimensional space[1] which are under the pressure of external particles. When the plasma is not dense and dissipative processes are negligible, the *Vlasov's approach*[2] to the Boltzmann equation is valid. We assume that this is the case and, thereby, we consider the Boltzmann equation for the one-particle distribution function in Vlasov's approximation of self-consistent fields:

$$\frac{\partial f(r,p,t)}{\partial t} + v\,\frac{\partial f(r,p,t)}{\partial r} + F(r)\,\frac{\partial f(r,p,t)}{\partial p} = 0 \tag{1}$$

where we adopted the radial symmetry of the problem and where

$$F(r) = -\frac{\partial V_2(r)}{\partial r} + q\,E = -\frac{\partial V_2(r)}{\partial r} - q\,\frac{\partial \phi_0(r)}{\partial r} \tag{2}$$

is a force which consists of two terms arose from self-consistent electrical and external potentials:

$$V_2(r) = \frac{C}{2\,\pi\,R^2}\,e^Y\,f(\,r\,)\,. \tag{3}$$

Here $f(r)$ is some function of r and $E = -\,grad\,\phi_0(r)$ in Eq. (2) is a self-consistent electric field created by our charged particles. This field is assumed to be weak enough in order to justify a linear approximation for the distribution function. We assume also that the magnetic field is small and we neglect the magnetic filed in our calculations. The Vlasov's system of equations include Maxwell's equations for the electric field:

$$rot\,E = 0\,, \quad div\,E = 4\,\pi\,q\,n\,\int\,f(r,p,t)\,d^3\,p\,, \tag{4}$$

where n is a particle's density.

[1]We formulate the problem in 3 dimensions because of importance of longitudinal dimension in real high energy interactions. The radial symmetry of the problem in this formulation is preserved as well and in the following we assume that all vectors of interest are radial. Therefore, the vector notations will be use only when it will be need in.

[2]Vlasov's approach uses a self-consistent electromagneetic field taking in account both external and internal (produced by the plasma particles) fields.

2.1 Linear approximation: equilibrium state

We solve the Boltzmann equation Eq. (1) by approximation of distribution function with some linear correction to the equilibrium state

$$f(r,p,t) \,=\, f_0(r,p) \,+\, f_1(r,p,t)\,, \;\; f_1 \,\ll\, f_0 \tag{5}$$

and some linear correction to the constant electrical field

$$E \,=\, E^0 \,+\, E^1\,. \tag{6}$$

We are looking for an equilibrium[3] state at the presence of external pressure in initial condition for the equation. Therefore, the Boltzmann equation which describes this equilibrium state has the following form:

$$v\,\frac{\partial\, f_0(r,p)}{\partial\, r} \,+\, F(r)\,\frac{\partial\, f_0(r,p)}{\partial\, p} \,=\, 0\,, \tag{7}$$

whose solution is the Boltzmann-Maxwell distribution function:

$$f_0(r,p) \,=\, \frac{1}{(\,2\,\pi\,k_B\,T\,)^{3/2}}\,e^{-\frac{p^2}{2mk_BT}\,-\,\frac{V_2(r)}{k_BT}\,-\,\frac{q\,\phi_0(r)}{k_BT}}\,. \tag{8}$$

The Maxwell equation for the electric field Eq. (4), therefore, reduces to the Poisson equation which has the following form in the limit of high temperatures (large kinetic energies of the particles in comparison to the potential energy):

$$-\,\Delta\phi_0(r) \,=\, 4\,\pi\,q\,n\,\left(1\,-\,\frac{V_2(r)}{k_BT}\,-\,\frac{q\,\phi_0(r)}{k_BT}\right) \tag{9}$$

or

$$\Delta\,\phi_0(r) \,-\, 4\,\pi\,q^2\,n\,\frac{\phi_0(r)}{k_BT} \,=\, -\,4\,\pi\,q\,n\,\left(1\,-\,\frac{V_2(r)}{k_BT}\right)\,. \tag{10}$$

We could rewrite it in the following form:

$$\Delta\,\phi_0(r) \,-\, \frac{\phi_0(r)}{r_D^2} \,=\, -\,Q_0 \,+\, \frac{V_2(r)}{q\,r_D^2}\,, \tag{11}$$

with $Q_0 \,=\, \frac{4\,\pi\,q\,n}{N}$ as the charge density and

$$r_D^2 \,=\, \frac{k_B\,T}{4\,\pi\,q^2\,n} \tag{12}$$

as the Debye length. Taking, as simplest example, the potential Eq. (??) in form of Eq. (??)

$$V_2(r) \,=\, \frac{C}{2\,\pi\,R^2}\,e^Y\,\theta\,(R^2\,-\,r^2\,)\,, \tag{13}$$

we are looking for two solutions in two different regions:

[3] In general, this state is quasi-equilibrium, the external pressure acts during finite period of time. Nevertheless, we assume, that this time period is long enough and we could consider this state as an equilibrium one during the time of system's evolution.

1. Region where $r > R$. In this region the potential Eq. (??) is zero and we obtain the usual screening equation for the electric potential:

$$\Delta \phi_0(r) - \frac{\phi_0(r)}{r_D^2} + Q_0 = 0 \,, \tag{14}$$

whose solution is

$$\phi_0(r) = Q_0 \, r_D^2 + C_0 \, \frac{e^{-r/r_D}}{r} \,. \tag{15}$$

2. Region where $r < R$. In this region we have

$$\Delta \phi_0(r) - \frac{\phi_0(r)}{r_D^2} + Q_0 - \frac{C}{2\pi R^2 q \, r_D^2} \, e^Y = 0 \tag{16}$$

with solution for the potential

$$\phi_0(r) = Q_0 \, r_D^2 + C_0' \, \frac{e^{-r/r_D}}{r} - \frac{C}{2\pi R^2 q} \, e^Y \,. \tag{17}$$

Matching the solutions at $r = R$ we obtain finally:

1. In the region where $r > R$

$$\phi_0(r) = Q_0 \, r_D^2 + C_0 \, \frac{e^{-r/r_D}}{r} \,. \tag{18}$$

2. In the region where $r < R$

$$\phi_0(r) = Q_0 \, r_D^2 + C_0 \, \frac{e^{-r/r_D}}{r} + \frac{C \, e^Y}{2\pi R^2 q} \left(\frac{R}{r} \, e^{\frac{R-r}{r_D}} - 1 \right) = Q_0' \, r_D^2 + C_0 \, \frac{e^{-r/r_D}}{r} \,. \tag{19}$$

with

$$Q_0' = Q_0 + \frac{C \, e^Y}{2\pi R^2 r_D^2 q} \left(\frac{R}{r} \, e^{\frac{R-r}{r_D}} - 1 \right) \tag{20}$$

as new non-screened charge arose due the external pressure. The value of C_0 is determined by some additional boundary conditions which we do not consider in the problem.

The Debye length, Eq. (12), depends on the particles density n and we could fefine the plasma's parameter as

$$\mu = \frac{1}{n \int^{r_D(n)} d^3x \int d^3p \, f_0(r,p)} \ll 1 \,. \tag{21}$$

Requiring the validity of this parameter in both cases of presence and absence of external field V_2 we obtain, that in the case of the existing of external pressure the Debye radius is smaller than in the case of the absence of the pressure:

$$(r_D(Y))_{V_2 \neq 0} < (r_D)_{V_2 = 0} \,. \tag{22}$$

It is known, see [9], that the hydrodynamic description of the process is possible when the characteristic external parameter of the system, in our case it is R, is larger than the r_D:

$$R > r_D \,. \tag{23}$$

Due to the external pressure the Debye radius decreases, and, therefore, it is possible that we arrive to the inequality Eq. (23) at large enough Y.

Now, in the linear approximation over $f_0(r,p)$, we have the Boltzmann equation

$$\frac{\partial f_1(r,p,t)}{\partial t} + v\,\frac{\partial f_1(r,p,t)}{\partial r} + \left(-\,\frac{\partial V_2(r)}{\partial r} + q\,E^0 \right) \frac{\partial f_1(r,p,t)}{\partial p} + q\,E^1\,\frac{\partial f_0(r,p)}{\partial p} = 0 \tag{24}$$

together with the Maxwell's equations for the electric field:

$$rot\,E^1 = 0\,, \quad div\,E^1 = 4\,\pi\,q\,n \int f_1(r,p,t)\,d^3\,p\,, \tag{25}$$

where

$$E^0 = -\,grad\,\phi_0(r) \tag{26}$$

with $\phi_0(r)$ given by Eq. (18) and Eq. (19).

2.2 Deviation from the equilibrium state

Proceeding with the equation Eq. (5) we perform a following substitution:

$$f_1(r,p,t) = \bar{f}_1(r,p,t)\,f_0(r,p) \tag{27}$$

with $f_0(r,p)$ from Eq. (8). Rewriting Eq. (24) we obtain:

$$\frac{\partial \bar{f}_1(r,p,t)}{\partial t} + \frac{p}{m}\,\frac{\partial \bar{f}_1(r,p,t)}{\partial r} + F\,\frac{\partial \bar{f}_1(r,p,t)}{\partial p} - \frac{q\,E^1}{m\,k_B\,T}\,p = 0\,, \tag{28}$$

where F is given by Eq. (2). We consider a linear approximation over the equilibrium distribution, therefore, we could write the external force in the equation in following form

$$F(r) \to \bar{F} = F(\bar{r}) = -\left(\frac{\partial V_2(r)}{\partial r} \right)_{r=\bar{r}} + q\,E^0(\bar{r})\,. \tag{29}$$

with r

$$\bar{r} = \frac{\int r\,f_0(r,p)\,d^3x\,d^3p}{\int f_0(r,p)\,d^3x\,d^3p}\,. \tag{30}$$

Thereby we have the following equation for the $\bar{f}_1(r,p,t)$ distribution function:

$$\frac{\partial \bar{f}_1(r,p,t)}{\partial t} + \frac{p}{m}\,\frac{\partial \bar{f}_1(r,p,t)}{\partial r} + \bar{F}\,\frac{\partial \bar{f}_1(r,p,t)}{\partial p} - \frac{q\,E^1}{m\,k_B\,T}\,p = 0 \tag{31}$$

Performing Fourier transform of $\bar{f}_1(r,p,t)$ and $E^1(r,t)$

$$\bar{f}_1(r,p,t) \;=\; \int \frac{d\omega}{2\,\pi} \int \frac{d^3k}{(2\,\pi)^3}\, e^{-i\omega\, t + i\, r\, k}\, \phi_0(k,p,\omega) \tag{32}$$

$$E^1(r,t) \;=\; \int \frac{d\omega}{2\,\pi} \int \frac{d^3k}{(2\,\pi)^3}\, e^{-i\omega\, t + i\, r\, k}\, \tilde{E}^1(k,\omega) \tag{33}$$

we obtain finally a nonlinear differential equation of the first order over momenta:

$$\frac{\partial\,\phi_0}{\partial\,p} \,+\, a\,p\,\phi_0 \,-\, b\,\phi_0 \,-\, c\,p \,=\, 0\,. \tag{34}$$

Here

$$a \,=\, \frac{ik}{m\,\bar{F}}\,,\;\; b \,=\, \frac{i\,\omega}{\bar{F}}\,,\;\; c \,=\, \frac{q\,\tilde{E}^1}{m\,k_B\,T\,\bar{F}}\,. \tag{35}$$

The solution of this equation, with additional condition that at $p \,\rightarrow\, 0\,(T \,\rightarrow\, 0\,)$ the solution is static, i.e. $\phi_0(k,p \,=\, 0\,,\omega) \,=\, 0$, is the following function:

$$\phi_0(k,p,\omega) \;=\; c\,e^{-a\,p^2\,/\,2 + b\,p} \int_0^p p^{'}\, e^{a\,p^{'2}\,/\,2 - b\,p^{'}}\, dp^{'}\,. \tag{36}$$

Thereby we obtain for the correction to the equilibrium state:

$$\phi_0(k,p,\omega) \;=\; \frac{q\,\tilde{E}^1}{m\,k_B\,T\,\bar{F}}\, e^{-a\,p^2\,/\,2 + b\,p} \int_0^p p^{'}\, e^{a\,p^{'2}\,/\,2 - b\,p^{'}}\, dp^{'}\,. \tag{37}$$

We see, that this correction is suppressed in comparison to the distribution function Eq. (8) by the factor $\frac{q\,\tilde{E}^1}{\bar{F}}$. Substituting this expression back in Eq. (32), we have:

$$\bar{f}_1(r,p,t) \;=\; \frac{q}{m\,k_B\,T\,\bar{F}} \int_0^p dp^{'}\,p^{'} \int \frac{d\omega}{2\,\pi} \int \frac{d^3k}{(2\,\pi)^3}\, \tilde{E}^1(k,\omega)\, e^{-i\omega\left(t + \frac{p^{'}}{\bar{F}} - \frac{p}{\bar{F}}\right) + i\,k\left(r + \frac{p^{'2}}{2\,m\,\bar{F}} - \frac{p^2}{2\,m\,\bar{F}}\right)}\,. \tag{38}$$

Performing Fourier transform again, we obtain for our function in (r,t) representation:

$$\bar{f}_1(r,p,t) \;=\; \frac{q}{m\,k_B\,T\,\bar{F}} \int_0^p dp^{'}\,p^{'}\,E^1(r \,+\, \frac{p^{'2} \,-\, p^2}{2\,m\,\bar{F}}\,, t \,+\, \frac{p^{'} \,-\, p}{\bar{F}}\,)\,. \tag{39}$$

This correction to the static distribution function determines also the non-static electric field which we consider in the next sub-section.

2.3 Non static electrical field of the gas

Using Eq. (4), we find the equation for the correction E^1 to the electric field:

$$div\,E^1(r,t) \;=\; \frac{4\,\pi\,q^2\,n}{m\,k_B\,T\,\bar{F}} \int d^3\,p \int_0^p dp^{'}\,p^{'}\,E^1(r \,+\, \frac{p^{'2} \,-\, p^2}{2\,m\,\bar{F}}\,, t \,+\, \frac{p^{'} \,-\, p}{\bar{F}}\,)\,f_0(r,p) \tag{40}$$

with with $f_0(r,p)$ from Eq. (8), see also Eq. (27). This equation is highly non-linear and non-local and, perhaps, a precise solution of the equation can be found only by numerical methods. In order to investigate approximate solutions of the equation, we perform Fourier transform of the functions in Eq. (40):

$$k\tilde{E}^1(k,\omega) = \frac{4\pi\, q^2\, n}{m\, k_B\, T\bar{F}} \int d^3p \int_0^p dp' p' \int \frac{d^3k'}{(2\pi)^3} \tilde{E}^1(k-k',\omega) e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i(k-k')\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \tilde{f}_0(k',p)\,, \tag{41}$$

where

$$f_0(r,p) \,=\, \int \frac{d^3k}{(2\,\pi)^3}\, e^{i\,r\,k}\, \tilde{f}_0(k,p)\,. \tag{42}$$

In the right hand side of the equation the oscillating integral over k' is not vanishing when

$$k' \,\propto\, \frac{2m\bar{F}}{p'^2 - p^2} \,\propto\, \frac{\bar{F}}{k_B\, T} \,\ll\, 1 \tag{43}$$

in our approximation of the weak external field. Therefore, in the region where

$$k \,>\, k' \tag{44}$$

in the first approximation over k' we have:

$$k\tilde{E}^1(k,\omega) \approx \tilde{E}^1(k,\omega) \frac{4\pi\, q^2\, n}{m\, k_B\, T\bar{F}} \int d^3p \int_0^p dp' p' \int \frac{d^3k'}{(2\pi)^3} e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i(k-k')\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \tilde{f}_0(k',p) -$$

$$- \frac{\partial\, \tilde{E}^1(k,\omega)}{\partial\, k} \frac{4\pi\, q^2\, n}{m\, k_B\, T\bar{F}} \int d^3p \int_0^p dp' p' \int \frac{d^3k'}{(2\pi)^3}\, k'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i(k-k')\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \tilde{f}_0(k',p) \tag{45}$$

or, simplifying the notations we obtain:

$$\frac{\partial\, \tilde{E}^1(k,\omega)}{\partial\, k} \,-\, \tilde{E}^1(k,\omega)\;(\, C_1(\omega,k) \,-\, C_2(\omega,k)\, k\,) \,=\, 0\,, \tag{46}$$

where

$$C_1(\omega,k) \,=\, \frac{\int d^3p \int_0^p dp' p'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i\,k\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} f_0(\frac{p^2-p'^2}{2m\bar{F}},p)}{\int d^3p \int_0^p dp' p' \int \frac{d^3k'}{(2\pi)^3}\, k'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i\,(k-k')\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \tilde{f}_0(k',p)} \tag{47}$$

and

$$C_2(\omega,\, k) \,=\, \frac{m\, k_B\, T\bar{F}}{4\pi\, q^2\, n \int d^3p \int_0^p dp' p' \int \frac{d^3k'}{(2\pi)^3}\, k'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i\,(k-k')\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \tilde{f}_0(k',p)}\,. \tag{48}$$

An integration of Eq. (45) gives

$$\tilde{E}^1(k,\omega) \,=\, C_0(\omega)\, e^{\int_0^k C_1(\omega,t)\, dt - \int_0^k C_2(\omega,t)\, t\, dt}\,. \tag{49}$$

We see, that the fluctuation of electric field in this region of k is suppressed by the large factor C_2 in the power of exponent.

In the opposite limit, when

$$k \sim k' \propto \frac{2m\bar{F}}{p'^2 - p^2} \propto \frac{\bar{F}}{k_B T} \ll 1 \tag{50}$$

we introduce new variable ϵ:

$$\epsilon = k - k'.$$

Equation Eq. (45) will acquire the following form:

$$k\tilde{E}^1(k,\omega) = \frac{4\pi q^2 n}{m k_B T \bar{F}} \int d^3p \int_0^p dp' p' \int \frac{d^3\epsilon}{(2\pi)^3} \tilde{E}^1(\epsilon,\omega) e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i\epsilon\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \tilde{f}_0(k-\epsilon,p). \tag{51}$$

Fourier transform of function Eq. (8) is

$$\tilde{f}_0(k,p) = \tilde{f}_0(k) f_0(p) = f_0(p) \int d^3r\, e^{-ir\,k} e^{-\frac{V_2(r)}{k_B T} - \frac{q\phi_0(r)}{k_B T}} \approx f_0(p) \left((2\pi)^3 \delta^3(k) - \frac{\tilde{V}_2(k)}{k_B T} - \frac{q\tilde{\phi}_0(k)}{k_B T} \right) \tag{52}$$

with

$$f_0(p) = \frac{1}{(2\pi k_B T)^{3/2}} e^{-\frac{p^2}{2mk_B T}}, \tag{53}$$

therefore, in the first approximation of expansion over ϵ we obtain:

$$\begin{aligned} \tilde{E}^1(k,\omega) = \; & \tilde{E}^1(k,\omega) \frac{4\pi q^2 n}{m k_B T \bar{F} k} \int d^3p\, f_0(p) \int_0^p dp' p'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i\,k\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} - \\ & - \tilde{E}^1(0,\omega) \frac{4\pi q^2 n \tilde{V}_2(k)}{m (k_B T)^2 \bar{F} k} \int d^3p\, f_0(p) \int_0^p dp' p'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)} \int^{\frac{\bar{F}}{k_B T}} \frac{d\epsilon\, \epsilon^2}{2\pi^2} - \\ & - \tilde{E}^1(0,\omega) \frac{4\pi q^3 n \tilde{\phi}_0(k)}{m (k_B T)^2 \bar{F} k} \int d^3p\, f_0(p) \int_0^p dp' p'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)} \int^{\frac{\bar{F}}{k_B T}} \frac{d\epsilon\, \epsilon^2}{2\pi^2}. \end{aligned} \tag{54}$$

Finally we obtain for our electric field:

$$\tilde{E}^1(k,\omega)\,\varepsilon = -\tilde{E}^1(0,\omega) \frac{2 q^2 n \bar{F}^2}{3\pi m (k_B T)^4 k} \left(\frac{\tilde{V}_2(k)}{k_B T} + \frac{q\tilde{\phi}_0(k)}{k_B T} \right) \int d^3p\, f_0(p) \int_0^p dp' p'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)} \tag{55}$$

where

$$\varepsilon = 1 - \frac{4\pi q^2 n}{m k_B T \bar{F} k} \int d^3p\, f_0(p) \int_0^p dp' p'\, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i\,k\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \tag{56}$$

is the dielectric constant of the problem.

3 Conclusion

In our paper we considered a toy model of particles interacting at high energy based on the effect of inertial confinement in plasma physics. There are few main components of the model which could be interesting for the case of real QCD interactions as well. First of all, this is the Pomeron like form of the potential for inter-particle interactions which depends on the relative rapidity of the interacting particles, second, it is a separation of the interacting particles on the "internal" and "external" ones and finally this is the reduction of non-equilibrium process to the approach for equilibrium state.

We know, that the QGP is an almost ideal liquid, see [1] and references therein, therefore, in our approach we assume that the process of gas compression is non-dissipative, i.e. dissipation could be manifested on the scales much smaller then the scales considered in the paper. In this case the compression of the "internal" gas area by "external" particles could be considered as an adiabatic process for the convergent to the center of area shock wave.

The assumption on the absence of the large-scale dissipative processes allows us to consider the Vlasov's approximation of the Boltzmann's equation for the self-consistent field for our system as an acceptable. In this case, we consider an influence of "external" particles as an effective field in the Vlasov equation. The way to solve this equation is well known, in linear approximation the equilibrium solution is given by Eq. (8) for the distribution function and by Eq. (10) for the electric field. As expected, the electrostatic field of the problem now depends on the form of external potential through Eq. (10), which also affects the value of Debye length of the problem. This length is smaller than in the case of external pressure absence, see Eq. (22), as expected. The further increase of the external pressure definitely will lead to the situation when the hydrodynamic approach of the process, following to [9], is possible. The condition for the possibility of that description is given by Eq. (23).

The non-static corrections to the equilibrium state for the distribution function and the electric field, given by Eq. (39) and Eq. (40) correspondingly, are very complicated and require additional analysis in another paper. Nevertheless, the results immediately show that both non-static corrections to the distribution function and correction to electric field are suppressed by the two large parameters of the problem: temperature and rapidity. This is a sign of absence of inconsistency in the calculations and of correct choose of equilibrium state in our calculations. Indeed, in the usual treatment of Vlasov equation we would have the ratio of two large parameters in the expression for non-static corrections, which would lead to contradiction of the definition of the corrections as "corrections" in the limit of large rapidity. The precise form and behavior of the tensor of the dielectric constant of the problem Eq. (56) also will be analyzed somewhere else.

Finally we would like to note, that the main purpose of the exploration of our toy model is an understanding of the main principles of the approach for the possible generalization of the approach for the case of quark-gluon plasma. The decoupling of fast particles from QGP and at the same time the influence of perturbative hard processes on the bulk of the particles of QGP could be, perhaps, described similarly in the proposed approach and may be useful in the calculations of different observables of high-energy scattering, such as multiplicities of produced particles for example, see [10]. Namely, we could try to describe the whole non-equilibrium process of QGP creation as an equilibrium process in some effective field related to the hard processes occurring at initial stages of plasma creation. This task is the next problem which we plan consider in our following studies.

Appendix A: gas of interacting particles

In this appendix we shortly remind the main facts concerning a virial expansion, see the detailed derivation in [11]. We consider a gas of N interacting particles each with mass m on a plane as a gas of hard disk with small thickness and radius r_0. The energy of the gas in the classical limit is given by the well known expression:

$$E(p,q) = \sum_{i=1}^{N} \frac{p_i^2}{2\,m} \, + U(b_1, ..., b_N) \tag{A.1}$$

where as usual the first term is the kinetic energy of N particles, U is a potential energy of their mutual interactions and $b_1, ..., b_N$ their coordinates. The grand partition function for this Hamiltonian is

$$Q(\mu, T, S) \,=\, e^{-\beta\,\Omega} \,=\, \sum_{N=0}^{\infty} e^{\beta\,\mu} \, \frac{1}{N!} \int \prod_{i=1}^{N} \frac{d^2\vec{p}_i \, d^2\vec{b}_i}{h^2} \, \exp(-\beta \sum_{i=1}^{N} \frac{p_i^2}{2\,m}\,) \, \exp(-\beta \, U(b_1, ..., b_N))\,. \tag{A.2}$$

where Ω is the grand potential of the problem. As usual, integrating over momenta, we obtain

$$Q(\mu, T, S) \,=\, e^{-\beta\,\Omega} \,=\, \sum_{N=0}^{\infty} \left(\frac{e^{\beta\,\mu}}{\lambda^2} \right)^N \, \frac{Z_N(S,\beta)}{N!} \tag{A.3}$$

and

$$Z_N(S, \beta) \,=\, \int ... \int \, d^2\vec{b}_1, ..., d^2\vec{b}_N \, \exp\left(-\beta \, U(b_1, ..., b_N)\right) \,. \tag{A.4}$$

Here λ is the De Broglie length of the quark corresponding to the average energy β:

$$\lambda \,=\, \left(\frac{h^2\,\beta}{2\,\pi\,m} \right)^{1/2} \,. \tag{A.5}$$

The potential of the problem is given by

$$\Omega \,=\, -\frac{1}{\beta}\, \ln \left(1 + \frac{e^{\beta\,\mu}}{\lambda} \, S \,+\, \frac{e^{2\,\beta\,\mu}}{2!\,\lambda^2} \int\int \, d^2\vec{b}_1 \, d^2\vec{b}_2 \, \exp\left(-\beta \, U(b_1, b_2)\,\right) \,+ ... \right) \,. \tag{A.6}$$

Here, we used

$$\int \, d^2\vec{b} \,=\, S \,= \pi \, R^2 \,, \tag{A.7}$$

where R^2 is the characteristic radius of the problem. In the following we will define and consider only pairwise interaction between the particles, namely we have

$$U(b_1, b_2) \,=\, U(|b_1 - b_2|) \,=\, U(b_{12}) \,=\, U(b) \,=\, U_{12} \tag{A.8}$$

and therefore, in the relative coordinates of the center mass we reduce the multiplicity of the integrated functions and obtain an additional S factor in the integrals:

$$\Omega \,=\, -P\,S \,=\, -\frac{1}{\beta}\,\ln\left(1 \,+\, S\,\frac{e^{\beta\,\mu}}{\lambda^2} \,+\, S\,\frac{e^{2\,\beta\,\mu}}{2!\,\lambda^4}\int\!\!\int d^2\vec{b}_{12}\,\exp\left(-\beta\,U_{12}\,\right) \,+ ...\right)\,. \qquad (A.9)$$

Introducing variable ζ

$$\zeta \,=\, \frac{e^{\beta\,\mu}}{\lambda^2} \qquad (A.10)$$

we obtain the expression for the potential in the form of the series in ζ

$$\Omega \,=\, -P\,S \,=\, -\frac{S}{\beta}\sum_{n=1}^{\infty}\frac{J_n}{n!}\,\zeta^n\,. \qquad (A.11)$$

We will take into account only two first terms of this series with the following J_1 and J_2:

$$J_1 \,=\, 1\,, \quad J_2 \,=\, \int\!\!\int d^2\vec{b}_{12}\,\left(\,\exp\left(-\beta\,U_{12}\,\right) \,-\, 1\,\right). \qquad (A.12)$$

The number of particles in this gas we obtain as usual

$$N = -\left(\frac{\partial\,\Omega}{\partial\,\mu}\right)_{T,S} \qquad (A.13)$$

and because $\partial\,\zeta\,/\,\partial\,\mu \,=\, \beta\,\zeta$ we finally have:

$$N \,=\, S\sum_{n=1}^{\infty}\frac{J_n}{(n-1)!}\,\zeta^n\,. \qquad (A.14)$$

Excluding from Eq. (A.11) and Eq. (A.14) the variable ζ, we obtain in the second order approximation the equation of state for our gas:

$$P \,=\, \frac{NT}{S} \,-\, \frac{N^2\,T}{2\,S^2}\,J_2\,, \qquad (A.15)$$

where $T = 1/\beta$.

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