TEMPERATURE DEPENDENCE OF VECTOR AND SCALAR SPIN CHIRALITIES OF Cu₃, V₃ NANOMAGNETS IN ROTATING MAGNETIC FIELD

Moisey I. Belinsky
School of Chemistry, Tel-Aviv University, Tel Aviv, Israel

belinski@post.tau.ac.il

Abstract

Temperature dependence of the vector \( \vec{r} \) and scalar \( \chi \) chiralities, orbital moment \( L_z \) and magnetization \( \vec{M} \) of the \( V_3 \), Cu₃ nanomagnets (NM) in the rotating magnetic field \( \vec{H}_1(\phi) \) of a given \( H_1 \) strength was considered. Different chirality behavior in the ground and excited frustrated spin states of trimer determine the temperature dependence of the net chirality of the system. Uniform polar rotation of the \( \vec{H}_1(\phi) \) field results in the continuous non-uniform \( \pm \gamma_\chi \) oscillations of the net chirality vector \( \vec{R}_\kappa \) of the system with respect to the Z-axis, correlated with the non-uniform \( \eta_\mu \) polar rotation of the net magnetization vector \( \vec{M} \) and simultaneous change of the magnitude and sign of the scalar chirality \( \chi \), which depend on the field strength \( H_1 \) and the temperature \( T_\kappa \). This temperature dependent behavior of the \( \vec{k}_{\tau_\kappa} \), \( \chi_{\tau_\kappa} \) chiralities and magnetization \( \vec{M}_{\tau_\kappa} \) at the temperature \( T_\kappa \) is governed by the correlation \( \chi_{\tau_\kappa} = 2(\vec{k}_{\tau_\kappa} \cdot \vec{M}_{\tau_\kappa}) \). An increase of the temperature results in the reduction of the magnitudes of the vector \( |\vec{k}_{\tau_\kappa}| \) and scalar \( \chi_{\tau_\kappa} \) chiralities and reduction of the \( \pm \gamma_\chi \) oscillation of the chirality vector \( \vec{k}_{\tau_\kappa} \). Manipulation of the vector \( \vec{k} \) and scalar \( \chi \) spin chiralities by the rotating magnetic field and variation of the temperature are of interest for the field control and application of the chirality of the \( V_3 \), Cu₃ NMs in the molecular based devices.

1. Introduction

Spin chirality, correlations between the chirality and magnetization, field manipulation and possible applications of chirality attract significant interest in molecular magnetism, nanomagnetism and magnetism of materials comprising trimeric clusters [1-8]. Antiferromagnetic (AFM) \( V_3 \), Cu₃ trimeric clusters [9-19], in which the Dzialoshinsky-Moriya [20, 21] (DM) coupling (\( H_{DM} = \sum \vec{D}_j (\vec{S}_j \times \vec{S}_j) \)) plays the central role in the formation of the energy spectrum and anisotropy of their magnetic and spectroscopic characteristics [9-19, 22-34], provide an opportunity to investigate [26-30] the effects of the field manipulation of the chirality, orbital and toroidal moments, frustration and spin arrangements in these DM trimers. Possible applications of chirality of the spin triangles in molecular-based devices, molecular spintronics and quantum computation have been proposed for the \( V_3 \), \{Cu₃\} NMs and other spin trimers [10a, 10b, 11a, 11b, 30-35] and triple quantum dots [36]. However, the questions of the applications of the scalar (vector) chirality of the spin trimers were discussed only in
general, without the consideration of the vector and scalar chiralities, the degeneracy, anisotropy and frustration of the energy levels of the $V_3$, $Cu_3$ NMs in zero field (ZF) and in an applied magnetic field. Upon the polar rotation of the field $\mathbf{H}_i(\phi)$, the GS scalar chirality $\chi_1$ (orbital moment $L_{z_1}$) is transformed simultaneously with the $\mathbf{k}_i$ and $\mathbf{M}_i$ vectors, in accord with the GS correlation $[28, 30] \chi_1 = 2(\mathbf{k}_i \cdot \mathbf{M}_i)$. The ground and excited frustrated states of the $V_3$, $Cu_3$ NMs are characterized by different correlated variations of the vector and scalar chiralities in magnetic field $[28, 30]$. The role of the vector and scalar chiralities of the excited states (ES) of the trimers and the $\kappa - \chi$ correlation in the net chirality of trimers and the temperature dependence of the system chirality have not been considered in the proposals of the applications of the scalar chirality of the $\{Cu_3\}$ NMs in molecular devices. The temperature dependence of the spin chirality of the DM trimers is important for the applications of chirality of these NMs. The temperature dependence of the chiralities of the $V_3$ $[10a]$, $\{Cu_3\} [11a]$ NMs with $|D_z| \sim 0.5K, J \sim 4-5K$ and of the $Cu_3$ complexes $[9, 12-19]$ with large $J$ and $D_z$ parameters ($D_z \sim 5-67K, J \sim 100-1150K$), in the rotating field, with an account of the different chiralities in the GS and ESs, has not been investigated. As will be shown, different behavior of the GS and ESs chiralities in the rotating $\mathbf{H}_i(\phi)$ field and the temperature population of the ESs upon an increase of the temperature result in the continuous non-uniform oscillation of the temperature-dependent chirality vector $\mathbf{k}$ of the system, the non-uniform rotation of the net magnetization vector $\mathbf{M}$, which is accompanied by the simultaneous continuous change of the magnitude and sign of the net scalar chirality $\chi$ (orbital moment $L_{z_1}$) of the system, that represents the field and temperature manipulation of chirality. The temperature dependence of the net chirality of the system shows that the chirality of the $Cu_3$ complexes with the large $D_z$ DM parameters have the advantage in application over the chirality of the $V_3, \{Cu_3\}$ NMs with the smaller $D_z$.

In multiferroics, the vector chirality $\mathbf{K}$ plays the principal role $[37]$. The rotation of the magnetic field $\mathbf{H}$ $[38-39]$ results in the generation, reversal and rotation of electric polarization vector $\mathbf{P}$ in multiferroics, which correlates $[37]$ with the behavior of the vector chirality $\mathbf{K}$. The chirality vector $\mathbf{K}$ can rotate by either $+90^\circ$ or $-90^\circ$ ($\pm 90^\circ$ flop of $\mathbf{K}$) in multiferroics $[38a, 38b]$ under the $90^\circ$ rotation of magnetic field $\mathbf{H}$ of a given strength. The $\mathbf{K}$ chirality vector is reversed by a $90^\circ$ rotation of $\mathbf{H}$ that demonstrates magnetic control of the vector chirality $[39]$. The flop ($90^\circ$-rotation) of the magnetization vector $\mathbf{M}$ in the rotating field is important for various magnetic systems, in particular, for multiferroics $[37]$. The peculiarities of the vector chirality $\mathbf{K}$ of the $Cu_3$, $V_3$ NMs in the rotating field, such as the $\mathbf{K}$ rotation, the possible flop of the chirality $\mathbf{K}$ vector, control of the spin chiralities by the rotating magnetic field and the variation of the temperature have not been considered yet. Following the multiferroics investigations $[38, 39]$, it is of interest to consider the control of the vector and scalar chiralities of the $Cu_3$,
V3 NMs by the rotation of the field $\hat{H}$ of a given strength. Manipulation of the spin chiralities by the rotating and tilted magnetic fields, by the variation of the temperature are of interest for the chirality application in molecular devices.

The aims of the paper are the following: (i) to consider the temperature dependence of the scalar $\chi$ and vector $\kappa$ chiralities of the V3, Cu3 NMs in the rotating magnetic field (the temperature and field manipulation of chirality); (ii) to describe the temperature-dependent non-uniform continuous oscillation of the net chirality vector $\kappa$, inhomogeneous continuous rotation of the net magnetization vector $M$ and the change of the magnitude (and sign) of the net scalar chirality $\chi$ of the frustrated V3, Cu3 NMs under the field rotation; (iii) to consider the correlations $\kappa - \chi - \dot{M}$ between the chiralities and magnetization of the system in the rotating field under an increase of the temperature.

2. Energy levels and spin chiralities of V3 NMs in rotating field

2.1 Spin Hamiltonian and spin chiralities of the spin levels

The energy spectrum, spin states, magnetism and spectroscopy of the equilateral (EQ) V3, Cu3 DM trimers is described by the spin Hamiltonian [9] ($J >> |D_z|$)

$$H_\ell = \sum \left\{ J (S_i \cdot S_j + 3/4) + D_z [S_i \times S_j]_z + \mu_B g_i S_i \cdot H \right\}. \tag{1}$$

The Heisenberg isotropic exchange $J$ coupling (the first term $H^0_{\ell}$) determines large $\Delta_\ell = 3J/2$ Heisenberg interval between the frustrated four-fold degenerate GS $2(S = 1/2)$ and non-frustrated achiral ES $S = 3/2$, $E^H_{I(IV)}[2(S = 1/2)] = 0$, $E^H(S = 3/2) = 3J/2$. The second term describes the out-of-plane $D_z$ DM coupling

$$H^z_{DM} = D_z \sum_{ij} [S_i \times S_j]_z. \tag{2}$$

$i,j = 12, 23, 31, D_\perp = 0$, which forms [9] the ZF splitting (ZFS), $2 \Delta = 2D_z = |D_z| \sqrt{3}$, between the ground $E^0_{I(II)}$ and excited $E^0_{III(IV)}$ ZF Kramers doublets of the frustrated $S = 1/2$ states of the EQ V3, Cu3 NMs, Fig. 1. The in-plane DMI $D_\perp$ is not considered in the model (1), (2), $D_\perp = 0$, since the out-of-plane $D_z$ DMI plays the central role in the formation of the spin chirality at ZF, low and intermediate magnetic fields. In the Zeeman coupling ($H_{Zeeman}$) with the external magnetic field (the third term in (1)), the anisotropy of the local g-factors was neglected, thus, e.g., $g_x = g_y = g_z = 1.959$ for V3 [10]. The trimer out-of-plane DM vector $D^z_\perp \parallel Z$ is oriented along the Z-axis perpendicular to the triangle XY plane, Fig.1, $D^z_\perp = \frac{1}{3} \sum D^z_i$. It is by now well established that the Heisenberg ($J$) plus DM ($D_z$) model $H_\ell \ (1) (D_z >> D_\perp)$, including distortions, describes the magnetism, EPR and MCD spectra of the Cu3, V3 trimers [9, 12-19, 28, 29] with the large DMI $D_z$ and $J$ parameters. The sign of the $D^z_{\perp+}$ DM parameters does not manifest itself in these experiments and usually is not considered.
The $-[+]$ sign of the DM parameter $D_\zeta^-$ [$D_\zeta^+$] determines the right-handed, $R$ (positive) [left-handed, $L$ (negative)] vector chirality, $\kappa_{1z}^R > 0$ [$\kappa_{1z}^L < 0$] of the GS of the discrete $V_3^{\text{R[L]}}$ and $Cu_3^{\text{R[L]}}$ NMs, $R[L] \leftrightarrow D_{\zeta}^{\pm1}$ [26-30], in accordance with the vector chirality of the spin triangles in 2D kagome lattices and triangular lattices [4, 7].

Fig. 1 Energy spectrum, vector and scalar chiralities of the frustrated $2(S = 1/2)$ states of the EQ $V_3^R$ trimer under the polar $\phi$ rotation of the magnetic field $\hat{H}_i(\phi)$, $J = 4.8K , D_\zeta^- = -0.5K , |\hat{H}_i| = 1T$. The initial GS axial vector chirality $\kappa_{1z}^R (\phi_0 = 0) = +1$ and magnetization $M_{1z}^- (0) = -1/2$ are shown in the scheme (1), $\chi_1 (0) = -1$ (see the text).

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Fig. 1 depicts the four $2(S = 1/2)$ frustrated $E_{1} - E_{\text{IV}} (|H_i|, \phi)$ spin states of the EQ $V_3^R$ trimer ($J = 4.8K$ [10a], $g = 1.96, D_\zeta^- = -0.5K$) and their chirality characterization under the uniform polar $\phi$-rotation of the applied magnetic field $\hat{H}_i(\phi)$ of the given strength $|\hat{H}_i| = H_i = 1T (H_{1T})$ in the vertical $XZ$ plane, $\phi = R(\hat{H}_i, Z)$, as shown in the scheme (1) in Fig. 1, $g_z = g_x , 0 \leq \phi \leq \pi , 2\Delta < 2h_{iz}$ for $H_{1T}^Z , h_i = g \mu_B H_i / 2$. The $E_{1,\text{IV}}(H_i, \phi)$ graphs describe the rotation dependence of the GS the ESs states, Fig. 1, $H_{1T}^Z$. For the achiral isotropic Heisenberg trimers $Cu_3^H, V_3^H$ ($D_{\zeta}^{\text{DM}} = 0$) with the isotropic $g$-factors, the Zeeman splitting $\Delta_{\zeta}^0 = 2h_{iz}$ is conserved under the uniform counterclockwise (CCW) polar $\phi$-rotation of the magnetic field $\hat{H}_i(\phi)$ of a given
strength $|\hat{H}_1| = H_1 = H_1 \sin \phi$, $H_{1z} = H_1 \cos \phi$) for the isotropic $E_{H(0)}^{I} = -h_n$, $E_{H(V)}^{I} = h_n$ levels, as shown by the corresponding dash-dotted dark yellow linear graphs in Fig. 1, $0 \leq \phi \leq \pi$, $h_n = g \mu_B H_{in} / 2$, $n = z, x, y$, $h_{iz} = h_{ix}, H_{i} < H_{LC}$. The operators $\hat{K}_n$, $n = z, x, y$ of the vector chirality and scalar chirality $\hat{C}$ (or $\hat{C}_z$) for the $S_i = 1/2$ trimers have the standard form [4, 7, 26a-29]:

\[ \hat{K}_n = (2/\sqrt{3}) \sum_{i,j=1,2,3} [\hat{S}_i \times \hat{S}_j]_n, \]

(3)

\[ \hat{C} = (4/\sqrt{3})(\hat{S}_1 \cdot [\hat{S}_2 \times \hat{S}_3]). \]

(4)

The matrix elements of the vector chirality operator $\hat{K}_n$ (3), and the scalar chirality operator $\hat{C}$ (4) in the $u_z (M_z)$ basis set are shown in [27, 28]. The right-handed (R) \{left-handed (L)\} vector chirality of the states of the spin trimers corresponds to $\kappa_z^{R} = +1 [\kappa_z^{L} = -1]$. The eigenfunctions [28] $u_z = u_z (M_z)$, $u_z^L = u_z (M_z^m)$ of the DM Hamiltonian $H_{DM} (2)$ and $H_i (H_c) (1)$ are characterized by the $\kappa_z^{R,L}$ vector and $\chi^z$ scalar chiralities in the field $H_{z}, PZ: u_z^R (M_z^+) = |\kappa_z^{R}, \chi_z^z, M_z^z >$, $u_z^L (M_z^m) = |\kappa_z^{L}, \chi_z^z, M_z^m >$. The ZF Kramers doublet $E_0^R (\kappa_z^{R}, \chi_z^z, M_z^z) \{ E_0^L (\kappa_z^{L}, \chi_z^z, M_z^m) \}$ with the ZF energy $d_z \{ -d_z \}$ is characterized by the $\kappa_z^{R} \{ \kappa_z^{L} \}$ vector chirality and zero scalar chirality.

The spin chiralities of the $Cu_3^{R,L}, V_3^{R,L} (D_3^{z+1})$ trimers in the rotating $\hat{H}_i (\phi)$ field are determined by the GS vector chirality $\kappa_i^R [\kappa_i^L]$ and corresponding GS scalar chirality $\chi_{iL} [\chi_{iL}]$ at very low temperature (LT) when the GS is mainly temperature populated, Figs. 1, 3. Simple level crossing (LC) of the GS $E_i (\beta_i, \phi)$ and ES $E_{\Pi} (\beta_i, \phi)$ of the EQ $V_3^{R}$ trimer in Fig.1 occurs at the $\phi = \pi / 2$ angle of the $\hat{H}_i (\phi)$ rotation, $0 \leq \phi \leq \pi$. The magnetic behavior is determined by the dimensionless magnetic field $\beta_i = g \mu_B |H_i| / |D_z| \sqrt{3}$. Different behavior of the $\kappa_i^R = \kappa_i^R (\beta_i, \phi)$ and $\kappa_i = \kappa_i (\beta_i, \phi)$ chirality vectors, as well as the $\hat{M}_i = \hat{M}_i (\beta_i, \phi)$ and $\hat{M}_{\Pi} = \hat{M}_{\Pi} (\beta_i, \phi)$ magnetization vectors and the $\chi_{iL}$ scalar chiralities in the $E_i (\phi, \beta_i)$ and $E_{\Pi} (\phi, \beta_i)$ states (Fig. 2), is important even at low $T_k$ temperature (LT), when these $I(\phi)$, $\Pi(\phi)$ lowest states are mainly temperature populated ($T_k$ designates here the temperature since $T$ is Tesla in Figures). The knowledge of the rotation behavior of the $\kappa_i, \chi$ chiralities and magnetization $\hat{M}$ in the ground and excited frustrated states of $V_3^{R}$, shown in Figs. 1, 2, makes it possible to consider the simultaneous transformations of the net $\kappa_i^R (\phi), \kappa (\phi)$ and $\hat{M}(\phi)$ characteristics of the EQ $V_3^{R}, Cu_3^{R}$ trimers, as a whole, at non-zero temperature $T_k \neq 0$, when the excited frustrated states are temperature populated.
2.2 Continuous non-uniform oscillation (rotation) of the net \( \mathbf{k}^R (\mathbf{M}) \) vector and transformation of the net scalar chirality \( \chi \) in the rotating field at \( T_K = 0.1K \)

In order to demonstrate the influence of the small increase of the temperature on the chirality and magnetization, Figs. 2-4 depict the rotation behavior of the net vector \( \mathbf{k}^R \) and scalar \( \chi \) chiralities and magnetization \( \mathbf{M} \) of the EQ \( V_3^R \) trimer (\( D_z = -0.5K \)) on an example of the temperature \( T_K = 0.1K \). The rotation behavior of the chirality vector \( \mathbf{k}^R = \mathbf{k}^R (\beta_\alpha, \phi, T_K) \) \( (\mathbf{k}^R_{\alpha x}) \) \{magnetization vector \( \mathbf{M} = \mathbf{M}([H_\alpha], \phi, T_K) (\mathbf{M}_{\alpha x}) \} \) in the vertical \( \text{XZ} \)-plane and the scalar chirality \( \chi = \chi_{\alpha x} = \chi([H], \phi, T_K) \) of the system at \( T_K = 0.1K \) were calculated, taking into account the temperature population \( \rho_N (T_K) \) of the frustrated ESs and different rotation behavior of their vector and scalar chiralities under the polar uniform \( \mathbf{H}_1 (\phi) \) rotation, \( H_1^{IT} \), Figs. 2-5. For the LTT \( T_K = 0.1K \), the chirality and magnetization of the two lowest \( I(\phi), \ II(\phi) \) levels, shown in Figs. 1, 5b, mainly determine the net \( \mathbf{k}^R \), \( \chi \) chiralities and magnetization \( \mathbf{M} \) of the \( V_3^R \) NM at \( T_K = 0.1K \) in Figs. 2-5 since only these states are thermally populated at this temperature for ZFS \( 2\Delta_{\text{DM}} = |D_z| \sqrt{3} \). The frustrated ESs \( \varepsilon_{\text{nit}} (\phi), \varepsilon_{\text{iy}} (\phi) \) of \( V_3^R \) in Fig. 1 are temperature populated at a higher temperature (see Figs. 5-7). The energy intervals between the four frustrated states, which possess different spin chiralities in Figs. 1 \( (H_1^{IT}), \ 5b \ (H_1^{0.5T}) \) determine the temperature behavior of the net vector and scalar chiralities of the system and the temperature manipulation of the spin chiralities, Figs. 4-7. The achiral ES \( S = 3/2 \) does not contribute to the chirality.

The components \( M_\alpha (\alpha = x, z) \) of the resulting magnetization vector \( \mathbf{M}(\phi) \) of the \( V_3^R \) \( (D_z) \) trimer as a function of the \( |H| \) field and temperature \( T_K \) are described by the standard expression \[ 1b] \]

\[
M_\alpha (H_\alpha, T) = (1/Z_N) \sum_N M_{N\alpha} \exp[-E_N(S)/k_BT_K],
\]

where \( M_{Nz}, M_{Nx} \) are the magnetization components of the \( E_N(S) \) state for the \( H_z, H_x \) direction of the applied field, for the \( N = I-IV \) frustrated \( 2(S = 1/2) \) \{non-frustrated ES \( S = 3/2, V-\text{VIII} \) states\}, \( Z_N = \sum_N \exp[-E_N(S)/k_BT_K] \) is the partition function. The temperature and field dependences of the \( \kappa^{\text{RII}} (|H|, \phi, T_K) \) \( (\alpha = x, z) \) components of the resulting \( \mathbf{k}^{\text{RII}} (|H|, \phi, T_K) \) chirality vector of the \( V_3^{\text{RII}} \) trimers in the \( \mathbf{H}(\phi) \) field, as well as the net scalar chirality \( \chi (|H|, \phi, T_K) \) of the system, are described by equations

\[
\kappa^{\text{RII}} (|H|, \phi, T_K) = (1/Z_N) \sum_N \kappa_{N\alpha} (|H|, \phi) \exp[-E_N(S_\alpha |H|, \phi)/k_BT_K], \quad (6)
\]

\[
\chi (|H|, \phi, T_K) = (1/Z_N) \sum_N \chi_N (|H|, \phi) \exp[-E_N(S_\alpha |H|, \phi)/k_BT_K]. \quad (7)
\]
where $\kappa_{N,\alpha}(|H|,\phi)$ is the $\alpha$-component of the chirality vector $\mathbf{k}_N(|H|,\phi)$ and $\chi_N(|H|,\phi)$ is the scalar chirality of the $N=I-IV$ frustrated state, see Figs.1, 5b, $\chi(|H|,\phi=\pi/2)=0$; $\kappa=\chi=0$ for the ES achiral $S=3/2$ state. The superscript $R$ of the right-handed cluster chirality is used in the net chirality $\kappa_R^R(|H|,\phi,T_K)$ since the GS $\mathbf{k}_1^R(\phi)$ ($\kappa_{1z}^R>0$) chirality determines the net vector chirality of the $V_3^R$ NM, as a whole. For the $V_3^L$ ($D^+_2$) trimer, the net cluster chirality is $\mathbf{k}_1^L(\phi)$ (left-handed) since the GS chirality is $\kappa_{1z}^L<0$.

Fig. 2 Dependence of the $\kappa_z^R$, $\kappa_x^R$ components and magnitude $|\mathbf{k}_R^R|$ of the net chirality vector $\mathbf{k}_R^R$, the magnitude of the net scalar chirality $\chi^+$ [orbital moment $L_z^+(\phi,T_K)$], the $M_z$, $M_x$ components and magnitude $|\mathbf{M}|$ of the magnetization vector $\mathbf{M}$ of the EQ $V_3^R$ trimer on the uniform rotation of the field $\mathbf{H}_1(\phi)$, $H_1^{IT}$, $T_K=0.1K$. 

Fig. 2 depicts the components $\kappa_z^R$, $\kappa_x^R$ of the net cluster chirality vector $\mathbf{k}_R^R$ of the variable $|\mathbf{k}_R^R|$ magnitude, which is shown by the open circles orange curve, as well as the $M_z$, $M_x$ components of the resulting magnetization vector $\mathbf{M}$ of the variable $|\mathbf{M}|$ magnitude and $\chi$ scalar chirality [orbital moment $L_z(\phi)=\chi(\phi)$] (solid light green curve), of the EQ $V_3^R$ trimer in the rotating $\mathbf{H}(\phi)$ field ($H_1^{IT}$) at $T_K=0.1K$. The temperature dependent $\kappa_z^R$ [$M_z$] component of the net chirality vector $\mathbf{k}_R^R$ [magnetization vector $\mathbf{M}$] of $V_3^R$ at $T_K=0.1K$ is very close to the $\kappa_{1z}$ [$M_{1z}$] components of the GS $\mathbf{k}_1^R$ [$\mathbf{M}_1$] vectors in Fig. 2. At the same time, the $\kappa_z^R$ [$M_z$] component of $\mathbf{k}_R^R$ [$\mathbf{M}$] (Fig. 2) differs from the corresponding GS $\kappa_{1z}$ [$GS M_{1z}$]
components of $\kappa_r^R$ $[\mathbf{M}_r]$. Fig. 2 depicts also the field-rotation dependence of the magnitude $|\kappa_r^R| = |\kappa_r^R|_{k_r=0.1K}$ of the net chirality vector $\kappa_r^R$ and the magnitude $|\mathbf{M}| = |\mathbf{M}|_{k_r=0.1K}$ of the net magnetization vector $\mathbf{M}$ of $V_3^R$ at $T_k = 0.1K$. The $|\kappa_r^R|$, $|\mathbf{M}|$ graphs at $T_k = 0.1K$ demonstrate significant non-linear decrease in the $\sim 50^\circ < \phi < 130^\circ$ range with the minimum at $\phi^\perp$, Fig. 2.

Continuous non-linear transformation of the net scalar chirality $\chi$ (orbital moment $L_z(T_k)$, $L_z^* = \chi^z$) of $V_3^R$ at $T_k = 0.1K$ is shown in Fig.2. Taking into account the ES chiralities and magnetization under an increase of the temperature $T_k$ leads to the continuous transformation of the magnetic characteristics and strong dependence of the magnitudes $|\kappa|$, $|\mathbf{M}|$ and $|\chi|$ of $V_3^R$ on the polar rotation of the field $\mathbf{H}_i(\phi)$ of the $|H_i| < H_{LC}$ strength in a wide range of the $\phi$ change, Figs. 2-4.

![Diagram](image)

Fig. 3 Continuous non-uniform $\gamma_k$ oscillation of the net chirality vector $\kappa_r^R$ and non-uniform $\eta_M$ polar rotation of the net magnetization vector $\mathbf{M}$ of the EQ $V_3^R$ trimer upon the uniform polar $\mathbf{H}_i(\phi)$ rotation at the temperature $T_k$ for $H_i = 1T$, $T_k = 0.1K$; $\gamma_k$, $\eta_M$ for $H_i = 0.5T$, $T_k = 0.1K$; $\gamma_k$, $\eta_M$ for $T_k = 0.01K$, $H_i = 1T$. The scheme (1) [(2)] shows the GS (ES) $\kappa_r^{I[II]}$, $\mathbf{M}_r^{I[II]}$ vectors and the net $\kappa_r^R$, $\mathbf{M}$ vectors at $T_k = 0.1K$ in the $0 \leq \phi < \pi/2$ [$\pi/2 \leq \phi < \pi$] range.

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In Fig. 3, the $\gamma_k$ graph (open red circles) [$\eta_M$ graph (open blue triangles)] shows the change of the oscillation [rotation] angle of the net $\kappa_r^R$ $[\mathbf{M}]$ vector of $V_3^R$ at $T_k = 0.1K$ under the polar uniform field $\mathbf{H}_i(\phi)$ rotation ($H_i^{1T}$), $\gamma_k = \gamma_k(H_i^{1T}, \phi, T_k)$ [$\gamma_k$],
\[ \eta_M = \eta_M(H_{1T}^I, \phi, T_K)[\eta_n]. \] The \( \gamma_K, \eta_M \) canting angles of the net \( \vec{r}_n, \vec{m} \) vectors of \( V_3^R \) at \( T_k \) are determined by equations: \( \tan \gamma_K = k^R_k / k^R_z \), and \( \tan \eta_M = M_z / M_x \), respectively, with the components \( k^R_x (k^R_z) \) and \( M_z (M_x) \) in Fig. 2. The angle dependences \( \gamma_i \) (solid magenta graph) and \( \eta_i \) (solid green) of the rotations of the GS vectors \( \vec{r}_i \) and \( \vec{m}_i \) are also shown for comparison in Fig. 3. The continuous graphs \( \gamma^* = \gamma^*(H_{1T}^I, \phi, T_K^*) \) and \( \eta^*_M = \eta^*_M(H_{1T}^I, \phi, T_K^*) \) for very low temperature \( T_K^* = 0.01K \) in Fig. 3 are close to the GS \( \gamma_i, \eta_i \) graphs, respectively, with the flop at \( \phi_i^\perp \) since only GS is mainly thermally populated at \( T_K^* \), \( H_{1T}^I \). The continuous graphs \( \gamma_i^*(\text{dashed wine}) \) and \( \eta^*_M \) (dash-dotted olive) in Fig. 3 show the oscillation and rotation of the net \( \vec{r}_n \) and \( \vec{m} \) vectors, respectively, of the EQ \( V_3^R \) trimer for \( H_{1T}^{0.5} \) (Fig. 5b) at \( \text{LT} T_K = 0.1K \). The \( \theta^0_M = \phi \) dash-dotted graph describes the \( \theta^0_M \) rotation of the GS \( \vec{m}_i^0 \) vector of the achiral Heisenberg \( V_3^H \) trimer.

The scheme (1) [(2)] of Fig. 3 depicts schematically the net two-dimensional chirality vector \( \vec{r}_n \) and the magnetization vector \( \vec{m} \) of \( V_3^R \) at \( \text{LT} T_K = 0.1K \) with the net canting angles \( \gamma_i \) (round orange arrow), \( \eta_i \) (round dark blue arrow) in the XZ-plane, respectively. The GS vectors \( \vec{r}_i \) \{ \( \vec{m}_i \) \} with the corresponding GS canting angles \( \gamma_i \) (round red arrow) \( \eta_i \) (round olive arrow), \( \gamma_i = \eta_i \), are shown in the scheme (1) [(2)] for the angle \( \phi \) close to \( \phi_i^\perp [\phi_i^\perp] \) in Fig. 3. The canting angles \( \gamma_i, \eta_i \) (\( \gamma_i = \eta_i \)) of the ES vectors \( \vec{k}_i, \vec{M}_i \) are also shown in the scheme (1), Fig. 3. The schemes (1), (2) and the \( \gamma_i, \eta_i \) graphs in Fig. 3 show that the canting angle \( \gamma_i(\phi_i) \) of the net chirality vector \( \vec{r}_n \) of \( V_3^R \) at \( T_K \) is less than the canting angle \( \gamma_i(\phi_i) \) of the GS chirality vector \( \vec{k}_1^R \) for the same canting angle \( \phi_i \) of the field \( \vec{H}_i(\phi_i) \), \( \gamma_i(\phi_i) < \gamma_i(\phi_i) \), in the wide range \( (\Delta_2 + \Delta_3) \) of the field \( \vec{H}_i(\phi) \) rotation in Fig. 3. At the same time, in these schemes (1), (2) and the \( \eta_i, \gamma_i \) graphs in Fig. 3, the canting angle \( \eta_i(\phi_i) \) of the net magnetization vector \( \vec{m} \) of \( V_3^R \) at \( T_K \), is more than the canting angle \( \eta_i(\phi_i) \) of the GS \( \vec{M}_1 \) vector at \( \phi_i \), \( \eta_i(\phi_i) > \eta_i(\phi_i) \), in the same range. The net \( \vec{r}_n \) and \( \vec{m} \) vectors, which are \( \parallel \) in the \( \Delta_i \) range, are nonparallel in the \( \Delta_2 \) range even at \( \text{LT} T_K = 0.1K \), \( \vec{r}_n \parallel \vec{m} \), as shown in the difference of the \( \gamma_i \) and \( \eta_i \) graphs in this \( \Delta_2 \) range, Fig. 3, \( \vec{r}_n \parallel \vec{H}_i, \vec{M} \parallel \vec{H}_i \).

The scheme (1) in Fig. 3 explains why even a small increase in temperature \( T_K \) (0.1K) and an account of the ES chirality vectors leads (i) to decrease of the \( \gamma_i \) angle of the non-uniform continuous oscillation of the net chirality vector \( \vec{k}_n \) of \( V_3^R \) at \( T_K = 0.1K \) in the \( \Delta_2 \) range in comparison with the \( \gamma_i \) canting of the GS \( \vec{k}_i^R \). This is
accompanied by (ii) an increase of the angle \( \eta_M \) of the non-uniform continuous rotation of the \( V_3^R \) net \( \vec{M} \) magnetization vector in comparison with the \( \eta_i \) canting of the GS \( \vec{M}_1 \) vector. The \( \gamma_\kappa \) and \( \eta_M \) continuous graphs in Fig. 3 coincide with the linear \( \gamma_i = \eta_i \) graph in the \( \Delta_1 \) range where \( k^R \parallel M^\dagger \). For \( H^{1T}_i \), the \( k^R_1 \) and \( k^R_\parallel \) vectors have opposite \( X \)-projections, as well as the positive \( \kappa^R_1 \), \( \kappa^R_\parallel \) Z-projections in the wide \( \Delta_2 + \Delta_3 \) range, Fig.2. This explains why an account of the ES \( k^R_\parallel \) vector chirality of the partially populated ES \( E_{S_{\parallel}} \) at \( LTT_k = 0.1K \) leads to the significant decrease [small change] of the net \( \kappa^R_1 \) \( [\kappa^R_\parallel] \) component, in comparison with the GS \( \kappa^R_1 \) \( [\kappa^R_\parallel] \) magnitude in Fig. 2. As a result, the temperature dependent canting angle \( \gamma_\kappa \) of the net \( k^R \) chirality vector is reduced in the schemes (1), (2), as shown also from the comparison of the \( \gamma_\kappa \) and \( \gamma_i \) graphs in the \( \Delta_2 + \Delta_3 \) range in Fig. 3.

At the same time, the GS \( \vec{M}_1 \) and ES \( \vec{M}_\parallel \) vectors have the same \( X \)-components \( (M^-_1, M^-_\parallel) \) and the opposite \( M^-_1, M^-_\parallel \) \( Z \)-components for \( H^{1T}_i \) in the range \( \Delta_2 + \Delta_3 \) in Fig. 3. For \( H^{0.5T}_1 \) (Fig. 5b), the \( \vec{M}_i \), \( \vec{M}_\parallel \) vectors have the same \( M^-_1 \), and \( M^-_\parallel \) components [opposite \( M^-_1 \) and \( M^-_\parallel \)] in the whole \( \phi \) range. This results in significant reduction [very small change] of the \( M^-_1 \) \( [M^-_\parallel] \) component of the net \( \vec{M} \) vector at \( T_k = 0.1K \) in the \( \Delta_2 + \Delta_3 \) range in comparison with the \( M^-_1 \) \( [M^-_1] \) components of the GS \( \vec{M}_1 \) vector, Fig. 2. This leads to an increase of the \( \eta_m \) angle of the continuous non-uniform rotation of the net magnetization vector \( \vec{M} \) in the \( \Delta_2 \) range in the scheme (1) in comparison with the \( \eta_i \) rotation angle of \( \vec{M}_i, \eta_i \leq \eta_m < \theta^0_M \), as shown from the \( \eta_M \), \( \theta^0_M \), \( \eta_i \) graphs in Fig. 3. The \( \vec{M}_i \) and \( \vec{M}_\parallel \) vectors in the scheme (2) in Fig. 3 analogously explain decrease of the \( \eta_m \) angle of the non-uniform continuous \( \vec{M} \) rotation in comparison with \( \eta_i, \theta^0_M < \eta_m (T_k) \leq \eta_i \), in the range \( \Delta_3 + \Delta_4 \).

The net chirality vector \( k^R (\phi^i, T_k) \) [magnetization vector \( M (\phi^i, T_k) \)] at \( \phi^i \) is directed along the \( Z \)-axis [\( X \)-axis], since \( \gamma_\kappa (\phi^i, T_k) = 0 \) \( [\eta_m (\phi^i, T_k) = 90^0 \)] in Fig. 3, \( k^R (\phi^i, T_k) \parallel M (\phi^i, T_k) \), the net scalar chirality vanishes at \( \phi^i \), \( \chi (\phi^i, T_k) = 0 \).

In more detail, under the uniform polar CCW rotation of \( H_i (\phi) (H^{1T}_i) \), Fig. 3, the net vector \( k^R \) of \( V_3^R \) at \( T_k = 0.1K \) experiences the following non-uniform \( \gamma_\kappa \) oscillation: (1) Relatively slow continuous non-uniform CCW rotation \( \gamma_{\kappa_{\phi}} (\phi) \) \( \{ (\phi \leftrightarrow T_{\phi_{\downarrow}}^{1\leq}) \} \) up to the maximal left inclination \( +\gamma_{\kappa_{\phi}}^{\text{max}} (\phi) \) of \( k^R \) at the angle \( \phi_{\downarrow} \), \( \gamma_{\kappa_{\phi}}^{\text{max}} (\phi) = \gamma_{\kappa_{\phi}} (\phi) + \gamma_{\phi_{\downarrow}} (\phi) \). This slow rotation includes the uniform reduced \( \gamma_{\kappa_{\phi}}^{\text{S}} (\phi) \) CCW rotation of \( k^R \) \( ([k^R] \approx 1 \), Fig. 2) in the \( \Delta_1 \) range and the non-uniform continuous
CCW rotation $\gamma_{k}^{S}(\phi)$ of $\mathbf{k}^{R}$ up to the maximal tilt $+\gamma_{k}^{\text{max}}(\phi_{m})$ in the $\Delta_{3}$ range at $T_{K} = 0.1K$; $\gamma_{k}^{m}(\phi_{m})$ is less than the maximal $\gamma_{k}^{m}$ tilt of the GS $\mathbf{k}_{i}$ vector at $\phi_{1}^{-}$, Fig. 3. (2) Then, after the $\gamma_{k}^{m}(\phi_{m})$ maximum at $\phi_{m}$, the $\gamma_{k}$ graph in Fig. 3 shows fast non-linear CW rotation $-\gamma_{2\beta}^{\text{fast}}(\phi)$ of the $\mathbf{k}^{R}$ vector of the decreasing $|\mathbf{k}^{R}|$ magnitude (Fig. 2) \{($\leftarrow\rightarrow_{2\beta}^{\text{fast}}$) \} against the field CCW rotation. The total angle $|\Sigma_{2\beta}^{\text{fast}}(\phi)|$ of the fast CW (right) continuous rotation $-\gamma_{2\beta}^{\text{fast}}$ of $\mathbf{k}^{R}$ (Fig. 3) coincides in magnitude with the maximal angle $+\gamma_{k}^{\text{max}}(\phi_{m})$ of the more slower CCW (left) continuous rotation, $\gamma_{k}^{\text{max}}(\phi_{m}) = |\Sigma_{2\beta}^{\text{max}}(\phi)|$. The slower $\gamma_{1\alpha}^{\text{slow}}(\phi)$, ($\leftarrow\rightarrow_{1\alpha}^{\text{slow}}$) \{faster $\gamma_{2\beta}^{\text{fast}}(\phi)$, ($\leftarrow\rightarrow_{2\beta}^{\text{fast}}$) \} $\mathbf{k}^{R}$ rotation at $T_{K}$ is designated by the $\leftarrow$ \{ $\rightarrow$ \} horizontal arrow. (3) Under the consequent uniform $\mathbf{H}_{1}(\phi)$ polar rotation in the $\Delta_{3} + \Delta_{4}$ range, the $\gamma_{k}$ graph in Fig. 3 shows fast continuous $-\gamma_{3\beta}^{\text{slow}}(\phi)(-\gamma_{3\beta}^{\text{fast}}(\phi))$ CW rotation \{( $\leftarrow\rightarrow_{3\beta}^{\text{slow}}$) \} of $\mathbf{k}^{R}$ in the $\Delta_{3}$ range up to maximal right inclination $-\gamma_{3\beta}^{\text{max}}(\phi_{m}) = -\gamma_{k}^{m}$ of $\mathbf{k}^{R}(\phi_{m})Z$ at $\phi_{m}$ in Fig. 3, $|\Sigma_{3\beta}^{\text{max}}| \mid |\Sigma_{3\beta}^{\text{max}}| \mid |\gamma_{k}^{\text{max}}|$. (4) Then, in the range $\Delta_{3} + \Delta_{4}$, the $\mathbf{k}^{R}$ chirality vector exhibits the slower non-uniform $\gamma_{4\alpha}^{\text{max}}(\phi) = \gamma_{C}^{\text{max}}(\phi) + \gamma_{D}^{\text{max}}(\phi)$ left (CCW) rotation \{( $\leftarrow\rightarrow_{4\alpha}^{\text{slow}}$) \}, which includes the $\gamma_{C}^{\text{max}}(\phi)$ continuous inhomogeneous CCW rotation from the maximal right slope $-\gamma_{3\beta}^{\text{max}}(\phi_{m})Z$ and the continuous reduced uniform $\gamma_{D}^{\text{max}}(\phi)$ left $\mathbf{k}^{R}$ rotation.

As a result, the continuous uniform $\phi$ -rotation of $\mathbf{H}_{1}(\phi)(H_{1}^{\text{IT}})$ in the $\Delta_{3} + \Delta_{2}$, $\Delta_{3} + \Delta_{4}$ ranges results in the continuous non-uniform oscillation $\pm\mathbf{V}_{3}^{R}$ net chirality vector $\mathbf{k}^{R}$ of the variable $|\mathbf{k}^{R}|$ magnitude with respect to the Z-axis (Figs. 2, 3), which includes the continuous non-uniform slower CCW $\gamma_{1\alpha}^{\text{slow}}(\phi)$ rotation, ($\leftarrow\rightarrow_{1\alpha}^{\text{slow}}$), the faster CW $\gamma_{2\beta}^{\text{fast}}(\phi)$ and $\gamma_{2\beta}^{\text{fast}}(\phi)$ continuous rotations, \{( $\leftarrow\rightarrow_{2\beta}^{\text{fast}}$) \} and \{( $\leftarrow\rightarrow_{2\beta}^{\text{fast}}$) \}, and the continuous slower $\gamma_{4\alpha}^{\text{max}}(\phi)$ CCW rotation \{( $\leftarrow\rightarrow_{4\alpha}^{\text{slow}}$) \}. Figs. 2, 3 describe this continuous $\pm\mathbf{V}$ non-uniform oscillation

$$\Gamma_{k}^{\mathbf{R}} = [\mathbf{S}_{1\alpha}^{\gamma}(\phi) + \mathbf{V}_{2\beta}^{\gamma}(\phi) + \mathbf{V}_{2\beta}^{\gamma}(\phi) + \mathbf{V}_{4\alpha}^{\gamma}(\phi)],$$

(8)

of the net chirality vector $\mathbf{R}_{3}$ of $\mathbf{V}_{3}^{R}$ at $LT_{K} = 0.1K$ under the continuous uniform polar rotation of the $\mathbf{H}_{1}(\phi)$ field in the $0 \leq \phi \leq \pi$ range. This $\pm\mathbf{V}$ oscillation with the acceleration and deceleration takes place in the upper $\frac{1}{2}(XZ)^{V}$ vertical half-plane.

The continuous $\gamma_{k}^{\mathbf{R}}(\eta_{M})$ graph in Fig. 3 describes the non-uniform oscillation [rotation] of the $\mathbf{k}^{R}(\gamma_{k}^{\mathbf{R}}, T_{K})$ \{ $\mathbf{M}(\eta_{M}, T_{K})$ \} vectors of $\mathbf{V}_{3}^{R}$ at $LT_{K} = 0.1K$ for the $\mathbf{H}_{1}(\phi)$ field strength $H_{1}^{0.5T}$. Taking into account of the temperature $T_{K} = 0.1K$ for $H_{1}^{0.5T}$ (Fig. 5b) leads to the different $\gamma_{k}$ and $\eta_{M}$ rotation behavior in the wide range of the field.
rotation in Fig. 3. The $\gamma'_{\kappa}$ graph demonstrates the smaller magnitude $\gamma'^{\text{max}}_{\kappa}$ of the maximal $\kappa^R$ canting for $H_1^{0.5T}$, in comparison with $\gamma^{\text{max}}_{\kappa}(\phi_m)$ at $\phi_m$ for $H_1^{1T}$. For the $\dot{\mathbf{M}}$ rotation, the $\eta^m_m(H_1^{0.5T})$ graph shows the larger deviation from the Heisenberg $\theta^H_m = \phi$ linear graph than in the $\eta^m_m(H_1^{1T})$ graph, since $|\beta'| < |\beta|$ corresponds to an increase of the DM $D_z$ parameter, which is represented in the larger range of the fast non-linear rotation of $\dot{\mathbf{M}}$ in Fig. 3. The interval of the joint uniform hindered rotation, $\gamma'_{\kappa} = \eta^m_m$, of the $\kappa^R(\gamma'_{\kappa})$ and $\dot{\mathbf{M}}(\eta^m_m)$ vectors for $H_1^{0.5T}$ in the left part of the $\gamma'_{\kappa} , \eta^m_m$ graphs in Fig. 3 is significantly less than the $\gamma'_{\kappa} = \eta^m_m$ part in the $\gamma'_{\kappa} , \eta^m_m$ graphs for $H_1^{1T}$. The decrease of $\beta'$ increases the non-uniform character of the $\gamma'_{\kappa} , \eta^m_m$ rotation, Fig. 3.

Fig. 4 The $\gamma'_{\kappa}$ non-uniform oscillation of the resulting chirality vector $\kappa^R$ and the $\eta^m_m$ non-uniform rotation of the net magnetization vector $\dot{\mathbf{M}}$ of the EQ $V_3^R$ trimer at temperature $T_K = 0.1K$ under the complete $2\pi$ rotation of the $\dot{\mathbf{H}}_1(\phi)$ field, $|H_1| = 1T$. The insert shows the simultaneous change of the magnitude and sign of the net scalar chirality $\chi^z$ and $|\kappa^R|$, $|\mathbf{M}|$ magnitudes of $V_3^R$. The $\theta^H_m - \eta^m_m$ graph shows the non-uniform oscillation of $\dot{\mathbf{M}}$ with respect to the uniform $\theta^H_m$ rotation of the magnetization vector $\dot{\mathbf{M}}_0$ of the achiral Heisenberg $V_1^H$ trimer.

This shows that under the uniform polar continuous CCW rotation of the $\dot{\mathbf{H}}_1(\phi)$ field ($H_1^{0.5T}, H_1^{1T}$) with the constant angular velocity $\omega_0$, the non-uniform $\pm \gamma$ oscillation (8) of the $V_3^R$ net chirality vector $\kappa^R(\phi, T_K)$ of the variable $|\kappa^R(\phi)|$ magnitude (Fig. 2) at LT $T_K = 0.1K$ in Fig. 3 occurs with the reduced maximal
angle magnitude, \( +\gamma_{\alpha\beta}^{\max} \leftrightarrow (-\gamma_{\alpha\beta}^{\max} \mathbf{Z}) \), \( \gamma_{\alpha\beta}^{m} = |\gamma_{\alpha\beta}^{m}| \), which depends on the strength \( H_{1} \) of the rotating field and the temperature.

The rotation of \( \hat{\mathbf{H}}_{1}(\phi) \) in the \( \pi \leq \phi \leq 2\pi \) range (Fig. 4) results in the continuous non-uniform oscillation \( \hat{\Gamma}_{\kappa} = \{\gamma_{s\kappa}(\phi) + \gamma_{f\beta}(\phi) + \gamma_{f\kappa}(\phi) + \gamma_{s\kappa}(\phi)\} \) of the variable net \( \hat{k}_{R} \) vector: \( (\uparrow \leftrightarrow \uparrow)_{s\kappa} \), \( (\uparrow \rightarrow \uparrow)_{f\beta} \), \( (\uparrow \rightarrow \mathbf{Z})_{f\kappa} \), \( (\uparrow \leftarrow \mathbf{Z})_{s\kappa} \). Fig. 4 shows that under the complete \((2\pi)\) uniform continuous \( \hat{\mathbf{H}}_{1}(\phi) \) rotation \( (H_{1}^{IT}) \), the net \( V_{3}^{R} \) chirality vector \( \hat{k}_{R} \) at \( T_{K} = 0.1K \) undergoes the two complete non-uniform \( \pm\gamma_{\kappa}^{\max} \) continuous oscillations \( \hat{\mathbf{f}}_{\kappa} = \hat{\Gamma}_{\kappa} + \hat{\Gamma}_{\kappa} \) in respect to the Z-axis in the upper vertical half-plane \( \frac{1}{2}(XZ)^{+}, \kappa_{R}(\phi) > 0 \).

The continuous non-uniform oscillation behavior \( \gamma_{\kappa} \) of the net \( \hat{k}_{R} \) vector is represented by the smoothed sawtooth continuous \( \gamma_{\kappa} \) graph, Fig. 4, which includes the slower CCW \( \gamma_{s\kappa}^{\max} (\phi) \) and faster CW \( \gamma_{f\kappa}^{\max} (\phi) \) continuous opposite rotations of \( \hat{k}_{R} \) with the acceleration-deceleration, \( \gamma_{\kappa} = \gamma_{s\kappa}^{\max} (\phi) + \gamma_{f\kappa}^{\max} (\phi) \). The maximal angle amplitude \( \pm\gamma_{\kappa}^{\max} (T_{K}) \) of the non-uniform oscillations of the net \( V_{3}^{R} \) vector \( \hat{k}_{R} \) at \( T_{K} = 0.1K \) is reduced in comparison with \( \gamma_{1\kappa}^{\max} \) of \( k_{R} \), Figs. 3, 4. The net \( V_{3}^{R} \) chirality vector \( \hat{k}_{R} \) at \( T_{K} \neq 0 \), which continuously \( \pm\gamma_{\kappa}^{\max} \) oscillates only in the upper half-plane \( \frac{1}{2}(XZ)^{+}(\kappa_{R} > 0 \text{ for } D_{\kappa}, \text{ Fig. } 2) \), cannot rotate by \( \pm90^{\circ} \) under the polar \( \Delta^{\pm} = \pm90^{\circ} \) rotation of \( \hat{\mathbf{H}}_{1} \), in contrast with multiferroics [38, 39]. The behavior of the \( V_{3}^{R} \) vector \( \hat{k}_{R} \) at the higher temperatures is shown in Figs. 6, 7.

3. Continuous non-uniform rotation of the net magnetization vector \( \hat{\mathbf{M}} \)

In the \( 0 \leq \phi \leq \pi (\Delta_{1} - \Delta_{1}) \) range of the uniform CCW polar \( \hat{\mathbf{H}}_{1}(\phi) \) rotation in Figs. 1-3, the net magnetization vector \( \hat{\mathbf{M}} = \hat{M}_{k} \) of the EQ \( V_{3}^{R} \) trimer at \( LTT_{K} = 0.1K \) exhibits the non-uniform continuous \( \eta_{M} \) CCW polar rotation, which is accompanied by the simultaneous chirality vector \( \hat{k}_{R} \) oscillation. (1) In the \( \Delta_{1} \) range, the \( \hat{\mathbf{M}} \) vector of the \( |\mathbf{M}| \approx 0.5 \) magnitude (Fig. 2) exhibits the continuous almost uniform \( \hat{\mathbf{M}}_{1}(\phi) \) (\( \eta_{1}(\phi) \)) polar CCW rotation in Fig. 3, which coincides with the GS \( \eta_{L}(\phi) \) rotation. This relatively slow \( \eta_{1} = \eta_{1}^{\max} \) rotation of \( \hat{\mathbf{M}} \) is reduced in comparison with the \( \theta_{\alpha}^{\max} = \phi \) graph of the uniform rotation of \( \hat{\mathbf{M}}_{1}(\phi) \) and \( \hat{\mathbf{H}}_{1}(\phi) \), \( \hat{\mathbf{M}} \uparrow \hat{\mathbf{H}}_{1} \). As discussed above, the \( \eta = \gamma \) part of the \( \eta_{M} \) and \( \gamma_{\kappa} \) continuous graphs shows the joint \( \eta_{M} = \gamma_{M}^{\alpha}(\phi) \) reduced rotation of the antiparallel vectors \( \hat{\mathbf{M}} \) and \( \hat{k}_{R} \) stuck together in the \( \Delta_{1} \) range. (2) In the \( \Delta_{2} \) range, the \( \eta_{M} \) part of \( \eta_{M} \) is larger than \( \gamma_{M}^{\alpha}(\phi) \), \( \eta_{M}(\phi) > \gamma_{M}^{\alpha}(\phi) \), \( \hat{\mathbf{M}} \uparrow \hat{k}_{R} \), Fig. 3. In particular, the maximal left tilt \( \gamma_{M}^{\alpha}(\phi) \) of the chirality vector \( \hat{k}_{R} \) at \( \phi_{m} \) in Fig. 3 is smaller than the canting angle \( \eta_{M}(\phi_{m}) \) of the magnetization vector \( \hat{\mathbf{M}}(\phi_{m}) \) at \( \phi_{m} \).

The \( \hat{\mathbf{M}} \) vector of the decreasing magnitude \( |\mathbf{M}| \) (Fig. 2) exhibits more fast continuous
CCW rotation $\eta^+ = \eta^+_b$ in the $\Delta_2$ range than $\eta^+$ in the $\Delta_1$ range in Fig. 3 up to the in-plane orientation $M^+ (\phi^+)$ for $\eta^+_b$ max $(\phi^+)$. The slower $\eta^+_{\text{slow}}$ and faster $\eta^+_{\text{fast}}$ CCW continuous polar rotations in the $\eta_M$ graph in Fig. 3 $(\eta^+_{\text{slow}} (\phi) + \eta^+_{\text{fast}} (\phi) = \pi / 2)$ demonstrate the non-uniform character of the continuous rotation of the net magnetization vector $\hat{M}$. As a result, in the $\Delta_1 + \Delta_2$ range, the continuous $\eta_M$ non-uniform rotation of $\hat{M}$ by $\pi / 2$ consists of (a) the slower continuous uniform $\eta^+_{\text{slow}} (\phi)$ $(\eta^+_{\text{slow}})$ rotation of $\hat{M}$ $(|M||=1/2)$ in the $\Delta_1$ range, Fig. 3, and then, (b) the faster $\eta^+_{\text{fast}} (\phi)$ $(\eta^+_{\text{fast}})$ rotation of the $\hat{M}$ vector of the variable $|M|$ magnitude in the $\Delta_2$ range, $\eta^+_M (\phi) + \eta^+_M (\phi) = \pi / 2$ (Figs. 3, 4).

In the consequent $\Delta_1 + \Delta_2 = \pi / 2$ range of the $\hat{H}_1 (\phi)$ rotation, the $\hat{M} (\eta, T_K)$ vector exhibits (3) the continuous fast CCW large $\eta^+_{\text{fast}} (\phi)$ non-linear rotation $(\eta^+_{\text{fast}})$ of the variable $\hat{M}$ vector in the $\Delta_1$ range characterized by the deceleration, $\hat{M} \rightarrow \hat{E}_K$, Figs. 3, 4. (4) Then, in the $\Delta_3$ range, the $\hat{M}$ vector demonstrates the slower CCW rotation $\eta^+_{\text{slow}} (\phi)$. In this range, the stuck together $M Z$ and $k^R Z$ vectors ($\hat{MP}_K^E$) of the same orientation exhibit the CCW rotation. The total non-uniform continuous $\hat{M}$ rotation in the $\Delta_3 + \Delta_4$ range is $\eta^+_{\text{slow}} (\phi) + \eta^+_{\text{slow}} (\phi) = \pi / 2$. As a result, the uniform continuous $\hat{H}_1 (\phi)$ field rotation in the range $0 \leq \phi \leq \pi$ leads to the non-uniform continuous polar rotation of the net magnetization vector $\hat{M}$ by $\pi$ angle $(M^+ (0) \rightarrow M^+ (\pi))$, Figs. 3, 4:

$$\Gamma^\eta (\phi, T_K) = \eta^+_{\text{slow}} (\phi) + \eta^+_{\text{fast}} (\phi) + \eta^+_{\text{slow}} (\phi) + \eta^+_{\text{fast}} (\phi) = \pi.$$

(9)

As shown above, this non-uniform continuous $\eta_{\hat{M}}$ CCW rotation (9) of $\hat{M}$ includes (i) the uniform continuous slower rotation ($\rightarrow \rightarrow$) $(\eta^+_{\text{slow}})$ in the $\Delta_1$ range, $|M||=1/2$, the two non-uniform continuous CCW faster rotations in the $\Delta_2$, $\Delta_3$ ranges: (ii) the accelerating $[] \rightarrow (\rightarrow)$ $(\eta^+_{\text{fast}})$ rotation, (iii) the decelerating $(Z \leftarrow (\rightarrow)$ $(\eta^+_{\text{slow}})$ rotation $(\eta^+_{\text{slow}})$, (iv) the uniform slower rotation $(\rightarrow Z \leftarrow \leftarrow)$ $(\eta^+_{\text{slow}})$ in the $\Delta_4$ range. The variable $|M|$ magnitude is shown in Fig. 2, 4. Under the $\pi \leq \phi \leq 2\pi$ field rotation, the $\hat{M}$ vector experiences the continuous non-uniform CCW rotation $\Gamma^\eta (\phi, T_K)$, which includes the slow $(\rightarrow \rightarrow)$ $(\eta^+_{\text{slow}})$, fast $(\leftarrow \leftarrow)$ $(\eta^+_{\text{fast}})$, fast $(\leftarrow \leftarrow)$ $(\eta^+_{\text{fast}})$, and slow $(\rightarrow \rightarrow)$ $(\eta^+_{\text{slow}})$ CCW rotations.

Fig. 4 shows that the complete $(2\pi)$ continuous uniform $\hat{H}_1 (\phi)$ polar rotation results in the complete $(2\pi)$ non-uniform continuous continuous $\Gamma^{\eta} (\phi, T_K) = \Gamma^{\eta} (\phi, T_K) + \Gamma^{\eta} (\phi, T_K) = 2\pi$ of the net magnetization vector $\hat{M}$ of $V^R$ at LT $T_K = 0.1K$, shown by the continuous $\eta_M$ graph (solid blue curve) in Fig. 4 of the smoothed sawtooth shape. This continuous $\eta_M$ graph includes the more slow rotations.
\( \eta_M \) of \( \mathbf{M} \) \((|\mathbf{M}| = 0.5) \), which are alternated by the accelerating-decelerating more rapid \( \eta_M \) rotations of \( \dot{\mathbf{M}} \) vector of the variable \(|\dot{\mathbf{M}}|\) magnitude (Fig. 4, Insert), \( \eta_M = \eta_M^{\text{flow}} + \eta_M^{\text{fast}} \). Continuous slower and faster rotations of \( \dot{\mathbf{M}} \) at \( T_K = 0.1K \), shown by \( \eta_M \) graph in Figs. 3, 4, can be compared with the \( \eta_m \) gradual plus sharp (flop) rotation of the GS \( \mathbf{M}_1 \) vector of \( V_3^R \) in Fig. 3.

The continuous \( \eta_M \) graph in Figs. 3, 4 oscillates with respect to the \( \theta_M^0 = \phi \) linear graph (tilted by \( \pi / 4 \)). The \( \theta_M^0 - \eta_M \) graph (dash-dotted gray) in Fig. 4 shows these non-uniform continuous oscillations of the net \( \dot{\mathbf{M}} \) vector, during its complete (2\( \pi \)) rotation, with respect to the uniform \( \theta_M^0 = \phi \) rotation of \( \dot{\mathbf{M}}_1(\phi) \), which means the deviations of the direction of the rotating vector \( \dot{\mathbf{M}} \) from the direction of the uniformly rotating vectors \( \dot{\mathbf{M}}_1(\phi)P\dot{\mathbf{H}}_1(\phi) \). \( |\theta_M^0 - \eta_M|_{\text{max}} < |\gamma^\text{max}| \). This confirms that the non-uniform continuous \( \dot{\mathbf{M}} \) rotation of the \( \dot{\mathbf{M}}_1 \) of the variable magnitude \(|\dot{\mathbf{M}}| \) at \( T_K \), which includes the slower and faster rotations with the acceleration and deceleration (Figs. 3, 4), differs significantly from the uniform \( \theta_M^0 = \phi \) continuous rotation of the magnetization vector \( \dot{\mathbf{M}}_1(\phi)\) \(|\dot{\mathbf{M}}_1(\phi)| = 1/2 \) of the achiral \((\kappa = \chi = 0)\) Heisenberg \( V_3^H \) trimer under the uniform continuous polar rotation of the field \( \dot{\mathbf{H}}_1(\phi) \) in Figs. 3, 4.

4. Variation of the scalar chirality \( \chi(T_K) \) correlated with the \( \dot{\mathbf{M}}(\eta, T_K) \) rotation and the \( k_1(\gamma, T_K) \) oscillation in the rotating field

Variation of the magnitudes and sign of the net scalar chirality \( \chi \) (orbital moment \( L_z \)) of \( V_3^R \) at \( LTT_K = 0.1K \), which is correlated with the change of the \( |\mathbf{k}|^R \) magnitudes, is shown in Fig. 2 and Insert in Fig. 4. The solid green \( \chi \) graph (Fig. 2) of the temperature-dependent net scalar chirality \( \chi = \chi_{T_K} = \chi(\beta, \phi, T_K) \) \{orbital moment \( L_z(T_K), L_\perp = \chi^\perp \) of \( V_3^R \) at \( T_K = 0.1K \) demonstrates the continuous transformation \[ [\chi^- (\phi = 0) = -1] \rightarrow [\chi^0 (\phi^+ = 0)] \rightarrow [\chi^+ (\phi^+ = 0)] \rightarrow [L_z^+(\phi) \rightarrow |L_\perp^+(\phi^+ = 0)] \rightarrow L_z(\phi) \] \} and differs from the GS \( \chi^\perp \) chirality (short dash-dotted cyan graph) with the sharp change at \( \phi_0^+ \). The \( \chi(\phi, T_K) \) graph shows that the \( \phi_0^+ = \phi_0^+ + \delta \phi \) deviation of the \( \dot{\mathbf{H}}_1(\phi) \) direction from the in-plane orientation \( H_{1z}^+(\phi^+) \) (Fig. 2) results in the significant change of the magnitude (and sign) of the \( \chi^m(\phi, T_K) \) chirality \(|L_z(T_K)|\) in Fig. 2, \( \chi^m(\phi) = 0 \). Figs. 2, 4 demonstrate the manipulation of the net scalar chirality \( \chi(\phi, T_K) \) (orbital moment \( L_z \)) of the \( V_3^R \) NM by the rotating \( \dot{\mathbf{H}}_1(\phi) \) field at \( LTT_K = 0.1K \).

Since the rotation behavior of the scalar \( \chi_N(\phi) \) and vector \( k_N(\phi) \), \( \dot{\mathbf{M}}_N(\phi) \) quantities is correlated in each \( N = 1-IV \) frustrated \( S = 1/2 \) state of \( V_3^R \),
\[
\chi_N = 2(\mathbf{r}_N \cdot \mathbf{M}_N),
\]
the non-linear change of the scalar chirality \( \chi_N(\phi, T_k) \) of the system at \( T_k = 0.1K \) (Figs. 2, 4) is strongly correlated with i) the inhomogeneous continuous \( \pm \gamma_k(T_k) \) oscillation of the corresponding net chirality vector \( \mathbf{r}^R(\gamma, T_k) \) and ii) the simultaneous non-uniform \( \eta_\pm(T_k) \) rotation of the net magnetization vector \( \mathbf{M}(\eta, T_k) \) of the system at LTT, Fig.4. As shown above, the directions of the net rotating vector \( \mathbf{M}(\eta, T_k) \) and oscillating vector \( \mathbf{r}^R(\gamma, T_k) \) coincide in the \( \Delta_1, \Delta_4 \) ranges of the uniform hindered rotations, and are different in the \( \Delta_2, \Delta_3 \) ranges of the non-linear behavior of \( \mathbf{r}^R, \mathbf{M} \) and \( \chi \), Figs. 2-4. However, in the rotating \( \mathbf{H}_l(\phi) \) field, the non-uniform rotation of the net magnetization vector \( \mathbf{M}(\eta, T_k) \), the non-uniform oscillation of the net chirality vector \( \mathbf{r}^{R\parallel}(\gamma, T_k) \), as well as the non-linear transformation of the net scalar chirality \( \chi(T_k) \), of the EQ \( \mathbf{V}_3^R \) \( \{ \mathbf{V}_3^L \} \) trimer in Figs. 2-4 are governed by the correlation
\[
\chi(\phi, T_k) = 2(\mathbf{r}^{R\parallel}(\gamma, T_k) \cdot \mathbf{M}(\eta, T_k)).
\]

The correlation (10) for the cluster \( \chi, \mathbf{r}^R, \mathbf{M} \) quantities in the rotating field with an account of the temperature population \( \rho_N(T_k) \) of the ESs is realized for all frustrated trimers. This consideration shows that the rotation behavior of the net chirality vector \( \mathbf{r}^R[\mathbf{R}^L] \), the scalar chirality \( \chi \) (net orbital moment \( L_\gamma \)) and magnetization vector \( \mathbf{M} \) of the \( \mathbf{V}_3^R \) \( \{ \mathbf{V}_3^L \} \) NM at the temperature \( T_k \) is strongly correlated. This means that the temperature and field dependence of the scalar chirality \( \chi \) (orbital moment \( L_\gamma \)) of the system is changed simultaneously with the vector chirality and magnetization (Figs. 2-7) and can not be considered independently of the \( \mathbf{k} \) and \( \mathbf{M} \) behavior, as it usually is considered.

5. Temperature and field manipulation of the spin chiralities in the rotating field

5.1 Chirality of the \( \mathbf{V}_3, \{ \mathbf{Cu}_3 \} \) NMs with the \( | D_z | \sim 0.5K \) parameters

Temperature dependence of the spin chirality is different for the \( \mathbf{V}_3, \{ \mathbf{Cu}_3 \} \) NMs with the \( | D_z | \sim 0.5K \) parameters (Figs. 5a, 5b, 7) and the \( \mathbf{Cu}_3^R \) complexes with the large DM parameters \( | D_z | = 5K \) (Figs. 6, 7). As shown in [28], the ground (\( | \kappa_1^R, \chi_1^-, M_1^z >, | \kappa_2^R, \chi_2^+, M_2^z > \) [excited (\( | \kappa_3^L, \chi_3^+, M_3^z >, | \kappa_4^L, \chi_4^+, M_4^z > \)]) doublet of the EQ \( \mathbf{V}_3^R \) trimer in ZF(\( H_1 = 0 \)), which are separated by the ZFS \( 2\Delta_{DM} = | D_z | \sqrt{3} \) (Fig. 5b), is characterized by zero net scalar chirality (orbital moment), \( \chi_{1,II}^{doublet} = \chi_{III,IV}^{doublelet} = 0 \), \( (\chi_- + \chi_+ = 0) \), \( L_{1,II,III,IV}^{doublelet} = 0 \), and the GS \( \kappa_2^R \) [ES \( \kappa_1^L \)] axial vector chirality, which is determined by \( D_z \). This shows that the system possesses zero scalar chirality \( \chi = 0 \) (zero orbital moment \( L_\gamma = 0 \)) in ZF. In the rotating magnetic field, \( H_1 \neq 0 \), the scalar chiralities \( \chi_1 \) and \( \chi_II \) of the lowest Zeeman states \( \epsilon_1^R(\phi), \epsilon_II^R(\phi) \) \{ \( \chi_III \) and \( \chi_{IV} \) of the excited \( \epsilon_III^L(\phi), \epsilon_{IV}^L(\phi) \) Zeeman states \} are opposite in the whole range of the field.
rotation, \( H_i < H_{\text{LC}} \), e.g. \( \chi_1^{[-]} , \chi_0^{[-]} , \chi_{\text{III}}^{[-]} , \chi_{\text{IV}}^{[-]} \) for \( \Delta_1 + \Delta_2 = \Delta_3 + \Delta_4 \). Fig. 5b, \( \chi(\mathbf{H}_1) = L_z(\mathbf{H}_1) = 0 \). For \( T_K = 0 \), the scalar chirality \( \chi \) of the system is determined by the GS \( \chi_1(T_K = 0) \) chirality, Figs. 1, 2. At \( \text{LT} T_K = 0.1 K \), the temperature population of the ES \( \varepsilon_{ii}(\phi) \) leads to the reduction of the net scalar chirality \( \chi \) of the system in the wide range \( \phi \approx \phi_1^* \) m60° in the vicinity of LC at \( \phi_1^* = \pi / 2 \) (Figs. 5a, 4) due to the partial compensation of the GS \( \chi_1^{[-]}[\chi_1^{[-]} \] chirality by the ES chirality \( \chi_{ii}^{[-]} \).

Fig. 5a demonstrates the temperature and field manipulation of the scalar chirality \( \chi_{tk} \) (orbital moment \( L_z(T_K) = \chi_{tk} \)) of the V_3^R \( [Cu_3^R] \) \( D_z = -0.5 K \) trimers in the rotating field \( \mathbf{H}_1(\phi) (H_{1,0}^{\text{ST}}) \) with an account of the temperature populations \( \rho_N(T_K) \) of all N = I-IV frustrated states and their spin chiralities shown in Fig. 5b. The GS chiralities \( |\kappa_1|, \chi_1 \) are also shown for the slightly distorted trimer in Fig. 5, \( \Delta J = 0.05 K \). The GS \( \chi_1^{[-]}(0), \chi_1, \chi_{tk} \) scalar chiralities (for \( T_K = 0.0,0.1 K, 0.5 K, 1 K, 2 K, 5 K \)) in Fig. 5a demonstrate significant reduction of the net scalar chirality \( \chi_{tk} \) [orbital moment \( L_z(T_K) \) ] of \( V_3^R \) when the \( T_K \) temperature increases. The increase of \( T_K \) up to \( T_K = 5 K \) results in the total reduction of the net scalar chirality \( \chi_{tk} \) (e.g., \( \chi_{5K} \approx 0 \), Fig. 5a) of the considered \( V_3^R \) \( (D_z = -0.5 K) \) NM with ZFS \( 2\Delta_{\text{DM}} \approx 0.87 K \) for all canting angles of the field rotation due to the compensation of the GS \( \chi_1 \) chirality by the chiralities of all ESs as their temperature population \( \rho_N(T_K) \) increases. For a given temperature \( T_K \), the maximal magnitude of the negative \( \chi_{tk}^{[-]} \) [positive \( \chi_{tk}^{[-]} \) ] net scalar chirality of the system takes place at \( H_{1}(\phi = 0) [H_{1}(\phi = \pi)] \) in Fig. 5a, since the Zeeman splitting is maximal for these field directions, Fig. 5b. At relatively high temperatures, which significantly exceeds the ZFS, \( 2\Delta_{\text{DM}} < T_K \), when all frustrated \( 2(S = 1/2) \) states are thermally populated, the net scalar chirality \( \chi_{tk} \) [orbital moment \( L_z(T_K) \) ] of the \( V_3^R \) DM trimer vanishes, Fig. 5a. The achiral excited \( S = 3/2 \) levels of \( V_3^R \) separated by the \( \Delta J = 3J / 2 \) interval from the frustrated states do not contribute to the chirality, \( H_1 < H_{\text{LC}} \). The manipulation of the scalar chirality \( \chi_{tk} \) (orbital moment \( L_z(T_K) \)) by the rotating field \( \mathbf{H}_1(\phi) \) and temperature \( T_K \) in Fig. 5a shows that the scalar chirality \( \chi_{tk} \) \( (L_z(T_K)) \) of the single EQ \( V_3^{R[I]} ) \) \( Cu_3^{R[I]} \) trimers with \( |D_z| \approx 0.5 K \) can be observed and used (i) at low temperatures and (ii) in magnetic field (preferably, longitudinal) when the non-degenerate Zeeman GS is separated from the ES by the \( 2h_1 \) interval.

Fig. 5a also depicts the reduction of the magnitude \( |\mathbf{k}_{tk}^{R}| \) of the oscillating net chirality vector \( \mathbf{k}_{tk}^{R} \) of this \( V_3^R \) NM in the rotating field when the temperature increases,
Fig. 5a Magnitudes of the vector $|\vec{K}_{\text{TR}}|$ and scalar $\chi_{\text{TR}}$ chiralities of the $V_3^R$ ($D_z = -0.5K$) trimer in the rotating $\vec{H}_i(\phi)$ field, $H_i = 0.5T$, $T_k = 0K, 0.1K, 0.5K, 1K, 2K, 5K$. Insert shows the reduction of the maximal angle $\gamma_{\text{TR}}^{\text{max}}$ of the $\pm \gamma_{\text{TR}}^{\text{max}}$ oscillation of the net chirality vector $\vec{K}_{\text{TR}}$ of the system in the rotating field as the temperature $T_k$ increases.

Fig. 5b Energy spectrum (frustrated $2(S = 1/2)$ states) of the $V_3^R$ NM under the polar uniform $\phi$ rotation of the magnetic field $\vec{H}_i(\phi)$ of the strength $|\vec{H}_i| = 0.5T$ (Fig. 5a), $J = 4.8K$, $D_z = -0.5K$. The vector chiralities $\vec{K}_{I}^R$, $\vec{K}_{II}^R$, $\vec{K}_{III}^R$, $\vec{K}_{IV}^R$, and the corresponding scalar chiralities $\chi_{I}^m$, $\chi_{II}^\pm$, $\chi_{III}^\pm$, $\chi_{IV}^m$ of the frustrated $E_{1,IV}(\epsilon_{1,IV})$ states of $V_3^R$ are also shown.

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$H_{1}^{0.5T}$, $T_{k} = 0.0.1K, 0.5K, 1K, 2K, 5K$. In this case, an increase in the temperature population $\rho_{N}(T_{k})$ of the excited $\varepsilon_{III}^{L}$, $\varepsilon_{IV}^{L}$ states in Fig. 5b, characterized by the L(left-handed) vector chirality $\Gamma_{L}^{R}$ (negative $\kappa_{L}^{R} > 0$ component) and separated by $2\Delta_{DM}$ from $\varepsilon_{III}^{R}$, leads to the partial compensation of the R(right-handed) vector chirality $\Gamma_{R}^{L}$ (positive $\kappa_{R}^{L} > 0$ component) of the lowest $\varepsilon_{III}^{R}$ doublet, Fig.5b. Since the $2\Delta_{DM}$ ZFS ($\parallel D_{z} \sqrt{3}$) is relatively small for this $V_{3}^{R}$ ($D_{z} = -0.5K$) trimer ($2\Delta_{DM} < 1K$), the increase of $T_{k}$ from 0.1K to 5K leads to a strong reduction of the $|\Gamma_{R}^{R}(T_{k})|$ magnitude of the system for all canting angles, Fig. 5a. The reduced magnitudes $|\Gamma_{R}^{R}(T_{k})| \approx 0.41(0.21)(0.07)$ of the $\Gamma_{R}^{R}$ vector for the temperatures $T_{k} = 1K(2K)[5K]$ do not depend on the angle of the field rotation. The temperature-induced reduction of the $|\Gamma_{R}^{R}(T_{k})|$ magnitude is less than the reduction of the corresponding maximal $\chi_{T_{k}}$ chirality, e.g., $|\Gamma_{R}^{R}(2K)[5K]| \approx 0.13(0.03)$, Fig. 5a. Insert in Fig. 5a depicts the $\gamma_{T_{k}}$ graphs of the non-uniform $\pm \gamma_{T_{k}}$ oscillations of $\Gamma_{R}^{R}(T_{k})$ in the rotating field for the temperatures $T_{k} = 0.1K, 0.5K, 1K$. The GS $\gamma_{1}(T_{k} = 0)$ and $\gamma_{0.1K}, \gamma_{0.5K}, \gamma_{1K}$ graphs demonstrate that the increase of the temperature $T_{k}$ also results in the large reduction of the maximal angle of the $\pm \gamma_{T_{k}}^{max}$ oscillations of the net chirality vector $\Gamma_{T_{k}}^{R}$ with respect to the Z-axis. Thus, e.g., the net chirality vector $\Gamma_{1K}^{R} = [\Gamma_{1K}^{R}, \Gamma_{2K}^{R}]$ of $V_{3}^{R}$, which possesses the reduced constant magnitude $|\Gamma_{1K}^{R}|$, $|\Gamma_{2K}^{R}|$ exhibits small $\pm \gamma_{T_{k}}^{max}$ non-uniform oscillations with the maximal angular amplitude $\gamma_{1K}^{max} \approx 1^\circ [0.3^\circ]$ at $T_{k} = 1K [2K]$ (Insert), while $\gamma_{0.1K}^{max} \approx 22^\circ$ for comparison, $H_{1}^{0.5T}$. This means that already at $T_{k} = 1K[2K]$, the reduced $|\Gamma_{1K}^{R}|$, $|\Gamma_{2K}^{R}|$ vector of the $V_{3}^{R}$ ($D_{z} = -0.5K$) NM is oriented practically $\parallel$ to the Z-axis (exhibits very small $\pm \gamma_{T_{k}}^{max}$ oscillations with respect to the Z-axis) in the rotating $\mathbf{H}_{1}(\phi)$ field, $H_{1}^{0.5T}$, the corresponding scalar chirality $\chi_{1K} \{ \chi_{2K} \}$ is reduced more strongly. For the $V_{3}$ ring of $V_{15}$ SMMs [23, 24] with small DM parameters $|D_{\phi}^{DM}| \approx 0.05 - 0.1K$, $J \approx 2.5$, and small ZFS $2\Delta_{DM}$, the spin chiralities is reduced at lower temperature $T_{k}$, Fig. 7.

5.2 Chirality of the Cu₃ trimers with the large D₃ parameters in the rotating field

Large number of the trimuclear $Cu_3$ (V₃) complexes are characterized [9, 12-19] by the large $D_{z}$, axial DM parameters, $|D_{z}| \sim 5 - 67$ K [14]. Their chirality and the possibility of its application in the molecular devices were not considered. Large $2\Delta_{DM}$ DM intervals between the ground doublet, $\varepsilon_{III}^{L}((\phi)$, and the excited doublet, $\varepsilon_{III,IV}^{L}((\phi)$ (Fig.5b), of the $2(S = 1/2)$ set in these frustrated trimers allow to investigate and use the spin chiralities of
these Cu₃, V₃ NMs even at relatively high temperatures, Fig. 7. Large $D_z$ parameter corresponds to small dimensionless field, $|\beta|<1$, even for high magnitudes $H_i$ of the rotating $\hat{H}_i(\phi)$ field. Figs. 6a, 6b show the rotation behavior of the resulting vector and scalar chiralities of the Cu₃R trimer with $D_z = -5K, J = 100K$, in the rotating field $\hat{H}_i(\phi), H_i = 2T, |\beta| \approx 0.3$. In this $|\beta|<1$ case, the excited levels $E_{\text{III,IV}}(\phi)$ of the $k_{\text{III,IV}}^{L}$ left vector chirality ($\chi_{\text{III}} \cdot \chi_{\text{IV}}^{m}$) in Fig. 5b are separated by the larger ZF interval $2\Delta_{\text{DM}} \approx 9K$ from the lowest $E_{\text{LII}}(\phi)$ two levels characterized by the right chirality vectors $k_{\text{LII}}^{R}$ ($\chi_{\text{I}}^{m}, \chi_{\text{II}}^{+}$). For these Cu₃ trimers with the larger ZFS $2\Delta_{\text{DM}}$, an increase of the temperature from $T_k = 0.1K$ to $T_k = 10K (20K)$ [50K] results in the reduction of the $|k_{\text{R}}|$ magnitude of the net $k_{\text{R}}^{R}$ chirality vector of the system from the maximal $|k_{\text{R}}^{R}| \approx 1$ magnitude to $|k_{\text{R}}^{R,0.2(50K)}| \approx 0.40(0.19)|0.06|$, Fig. 6a, 7, due to the increasing temperature population $\rho_{N}(T_k)$ of the $E_{\text{III,IV}}(\phi)$ excited levels of the opposite $k_{\text{LII}}^{L}$ left vector chirality upon an increase of $T_k$ and the resulting partial compensation of the GS $k_{\text{I}}^{R}$ right vector chirality in the net chirality vector $k_{\text{R}}^{R}$. Beginning from $T_k = 3K$, the reduced magnitudes $|k_{\text{R}}^{R}|$ in Fig. 6a do not depend practically on the $\hat{H}_i(\phi)$ field rotation. Fig. 6b demonstrates the change of the canting angle $\gamma_{\text{R}}$ of the net magnetization vector $k_{\text{R}}^{R}$ of this Cu₃ trimer in the rotating field as the temperature $T_k$ increases. An increase in $T_k$ leads to the reduction of the maximal angle $\pm \gamma_{\text{R}}^{\text{max}}$ of the non-uniform oscillations of the net vector $k_{\text{R}}^{R}$ with respect to the Z-axis, Fig. 6b. The $\gamma_{\text{R}}$ graph shows the complete reduction of the $\pm \gamma_{\text{R}}^{\text{max}}$ oscillation of the reduced $|k_{\text{R}}^{R}|$ vector at $T_k = 10K$. Fig. 6a ($H_i = 2T$) shows that the scalar chirality $\chi_{\text{R}}$ (orbital moment $L_z(T_k)$) of this Cu₃R, $D_z = -5K$ NM with the larger ZFS $2\Delta_{\text{DM}}$ is significant at $T_k = 0.5K, 1K, 2K, 3K$ and even at $T_k = 5K$ in comparison with the small $\chi_{\text{I}}$, scalar chirality of the V₃R, Cu₃R ($D_z = -0.5K$) trimers at $T_k = 1K$ ($H_i = 0.5T, 1T$) in Figs. 5-7. An increase in the $T_k$ temperature leads to the reduction of the scalar chirality $\chi_{\text{R}}$, Fig. 6a, 7. The scalar chirality $\chi_{\text{R}}$ ($\approx -0.05$) at $T_k = 10K$ in Fig. 6a, 7, which is practically almost reduced in comparison with $\chi_{\text{I}}$ ($\approx -1.0$) is much less than the reduced magnitude of the corresponding vector chirality $|k_{\text{R}}^{R}| \approx 0.40$ at the same temperature $T_k$.

The $\eta_{\text{R}}$ graphs in Fig. 6b for $T_k = 0.1K, 0.5K, 1K, 2K, 3K, 5K$ show that under an increase of $T_k$, these $\eta_{\text{R}}$ curves tend to the $\theta_{M}^{0}$ linear graph of the uniform rotation of the $\hat{M}_0(\phi)$ magnetization vector of the achiral Heisenberg V₃H trimer. The CCW $\eta_{\text{R}}$ rotation of the net magnetization vector $\hat{M}_{\text{R}}(\phi)$ of the Cu₃R ($D_z = -5K, J = 100K$) trimer is transformed from (i) the non-uniform $\eta_{\text{R}}$ rotation (e.g., $\eta_{0.1K}, \eta_{0.5K}, \eta_{1K}$) with significant continuous non-uniform oscillations with respect to the linear $\theta_{M}^{0}$ graph, which takes place.
Fig. 6. (a) Net scalar chiralities $\chi_{x_k}$ and magnitudes $|k_T^R|$ of the net vector chirality $k_T^R$ of the $Cu_3^R$ $(D_z = -5K, J = 100K)$ trimer in the rotating $H_i(\phi)$ field, $H_i = 2T$, for the given temperature $T_k, T_k = 0.1K; 0.5K; 1K; 2K; 3K; 5K; 10K; 20K; 50K$. (b) Reduction of the non-uniform $\pm k_T^R$ oscillations of the net chirality vector $k_T^R$ in Fig. 6a upon an increase of the temperature $T_k$. Non-uniform CCW rotation $\eta_{x_k}$ of the net magnetization vector $M_{x_k}$ of the system at the temperatures is also shown.

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at LT, to (ii) the continuous rotation (e.g., \( \eta_{5K} \)) very close to the \( \theta_M^0 \) graph of the uniform \( \vec{M}_0(\phi) \) rotation. The excited \( S = 3/2 \) state of \( Cu^R_{3} \) (\( D_z = -5K, J = 100K \)) with the large \( \Delta(S = 1/2 - S = 3/2) = 3J/2 + \Delta_{DM} - h_z \) interval contribute to the magnetization at higher temperatures.

Fig. 7 depicts the temperature dependence of the net right vector chirality \( \kappa^R_z(T_K, H_{1z}) \) \((\kappa^{a[b]}_z(H_{1z}))\) and the net negative scalar chirality \( \chi^{a,b,c}_z(T_K, H_{1z}) \) of the following trimers: \( Cu^R_3 \) (\( D_z = -5K, J = 100K \)) \((a)\), \( Cu^R_3 \), \( V^b_3 \) \((D_z = -0.5K, J = 4.8K) \) \((b)\), \( V^c_3 \) \((D_z = -0.5K, J = 2.5K) \) \((c)\), in the temperature range \( 0 \leq T_K \leq 10K \), in the longitudinal magnetic fields \( H_{1z} = 0.5T, 1T \), since the maximal magnitudes of the vector and scalar chiralities take place at \( H_{1z}(\phi = 0) = \|Z \) in Figs. 6a, 6b. Fig. 7 shows that the vector chirality \( \kappa^a_z(T_K, H_{1z}^{0.5T}) \) for \( D_z = -5K \) significantly exceeds \( \kappa^b_z(T_K, H_{1z}^{0.5T}) \) for \( D_z = -0.5K \), which, in turn, exceeds \( \kappa^c_z(T_K, H_{1z}^{0.5T}) \) for \( D_z = -0.1K \) at all temperatures. The magnitudes of the scalar chiralities \( \chi^{a,b,c}_z(T_K, H_{1z}) \) are reduced upon an increase of the temperature \( T_K \) more rapidly than the corresponding \( \kappa^R_z(T_K, H_{1z}) \) vector chiralities in Fig. 7, in accord with Figs.

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**Fig. 7** Temperature dependence of the vector chirality \( \kappa^{a[b]}_z(H_{1z}) = \kappa^R_z(T_K, H_{1z}) \) and scalar chirality \( \chi^{a[b]}_z(H_{1z}) = \chi(T_K, H_{1z}) \) of the \( Cu^R_3 \) trimers with \( D_z = -5K, J = 100K \) \((a)\) \([ Cu^R_3, V^R_3 \) NM \( D_z = -0.5K, J = 4.8K \) \((b)\) \], \( V^R_3 \) trimers \( D_z = -0.1K, J = 2.5K \) \((c)\) in the field \( H_{1z} = 0.5T, 1T \).

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6a, 6b, \( \chi(T_K, H_z) = 0 \). The magnitude of the scalar chirality \( \chi^a(T_K, H_{1z}^{0ST}) \) of \( Cu^a_5 \), \( D_z^a = -5 \text{ K} \) exceeds \( \chi^b(T_K, H_{1z}^{0ST}) \) for \( D_z^b = -0.5 \text{ K} \), which, in turn, exceeds \( \chi^c(T_K, H_{1z}^{0ST}) \) for \( D_z^c = -0.1 \text{ K} \) at all temperatures. The chiralities \( \chi^a(T_K, H_{1z}^{1T}) \) at \( H_{1z} = 1 \text{ T} \) and \( \chi^a(T_K, H_{1z}^{0ST}) \) at \( H_{1z} = 0.5 \text{ T} \) show significant increase of the scalar chirality upon an increase of the field magnitude \( H_{1z} \) (PZ), while the vector chiralities \( \kappa^e(T_K, H_{1z}^{0ST}) \) and \( \kappa^e(T_K, H_{1z}^{1T}) \) coincide.

The joint rotation behavior of the oscillating net chirality vector \( \mathbf{K}_R^{[3L]}(T_K) \), the variable net scalar chirality \( \chi(T_K) \) and rotating magnetization vector \( \mathbf{M}(T_K) \) of the frustrated \( V_3^{[3L]} \), \( Cu^{[3L]}_3 \) NMs in the rotating \( \mathbf{H}_1(\phi) \) field, \( H_1 < H_{LC} \), at the temperature \( T_K \) in Figs. 2-7 are connected by the correlation (10).

Figs. 6a, 6b, 7 demonstrate that the temperature-dependent resulting vector \( \mathbf{K}_R^{[3L]}(T_K) \) and scalar \( \chi(T_K) \) chiralities of the trinuclear \( Cu^{[3]}_3 \) DM complexes with the large \( D_z^{1[+]} \) DM and Heisenberg \( J \) parameters can be observed and used at the higher temperatures, in comparison with the spin chiralities of the \( V_3^{[3]} \), \( Cu^{[3]}_3 \) (\( |D_z| = 0.5 \text{ K} \) ) trimers in Figs. 4, 7 and \( V_3 \) ring of \( V_{15} \) SMM (\( |D_z| = 0.05 - 0.15 \text{ K} \) ). This shows the advantage of the \( Cu^{[3]}_3 \) trimers [9, 12-19] with the large \( D_z \) parameters over the \( V_3^{[3]} \) [10a], \( Cu^{[3]}_3 \) [10b] NMs with the smaller \( D_z \) in the possible application of their scalar \( \chi \) and vector \( \mathbf{k} \) chiralities (orbital moment \( L_z \) ) in molecular devices. Figs. 2-7 show the control (manipulation) of the vector and scalar chiralities of the \( V_3^{[R]} \), \( Cu^{[R]}_3 \) trimers by the variation of the temperature \( T_K \) and the strength of the rotating field \( \mathbf{H}_1(\phi) \).

6. Conclusion

Dependence of the vector \( \mathbf{k} \) and scalar \( \chi \) chiralities of the \( Cu^{[R]}_3 \), \( V_3^{[R]} \) (\( Cu^{[L]}_3 \), \( V^{[L]}_3 \) ) trimers in the rotating \( \mathbf{H}_1(\phi) \) field on the temperature, the correlation between the magnetization \( \mathbf{M} \), vector \( \mathbf{k} \) and scalar \( \chi \) chiralities of the system under an increase of the temperature have been considered. Uniform polar rotation of the \( \mathbf{H}_1(\phi) \) field of a given \( H_1 \) strength, \( |H_1| < H_{LC} \), results in (i) the continuous non-uniform \( \pm \gamma_\chi \) oscillations of the \( V_3^{[R]} \) net chirality vector \( \mathbf{K}_R^{[R]} \) of the variable \( |\mathbf{k}^{[R]}| \) magnitude with respect to the Z-axis, \( \mathbf{k}^{[R]}(\gamma) \subset \mathbf{H}_1(\phi) \) (Fig. 4), (ii) the non-uniform \( \eta_m \) polar rotation of the net magnetization vector \( \mathbf{M} \) of the system, \( \mathbf{M} \subset \mathbf{H}_1 \), \( \mathbf{M} \subset \mathbf{K}^{[R]} \), and (iii) the correlated non-linear change of the magnitudes and sign of the scalar chirality \( \chi \) (orbital moment \( L_z \) ), Figs. 2-7, which depend on the field strength \( |H_1| \) and the temperature \( T_K \). Continuous non-uniform \( \pm \gamma_\chi \) oscillations of the net chirality vector \( k^{[R]}_R \) of the \( V_3^{[R]} \) NM (the non-uniform \( \eta_m \) continuous rotation of
the magnetization vector $\hat{M}$ in the rotating $\hat{H}_r(\phi)$ field demonstrate the ranges of the slower and faster oscillations (rotation) with the acceleration and deceleration.

An increase of the temperature $T_k$, with an account of the different rotation behavior of the spin chiralities in the ground and excited states and the temperature population of the excited frustrated levels, results in (i) the decrease of the $|k^R_{tk}|$, $|\chi_{tk}|$ magnitudes of the net vector $k^R_{tk}$ and scalar $\chi_{tk}$ chiralities of the $V_3^R$, $Cu_3^R$ NMs, (ii) the reduction of the maximal $\gamma_{tk}^{\max}$ angle amplitude of the non-uniform $\pm \gamma_{tk}$ oscillation of $k^R_{tk}$, and (iii) the reduction of the $\theta_m^0 - \eta_m$ oscillation of the magnetization vector $\hat{M}_{tk}$ with respect to the graph $\theta_m^0$ of the uniform rotation of the magnetization vector $\hat{M}_0$ of the Heisenberg $V_3^H$ achiral trimer (Figs. 2-7). The magnitude $|\chi_{tk}|$ of the scalar chirality at the temperature $T_k$ is reduced more significantly than the $|k^R_{tk}|$ magnitude, Fig. 7.

Under the polar rotation of $\hat{H}_r(\phi)$, the temperature-dependent net vector chirality $k^R_{tk}$ and scalar chirality $\chi_{tk}$ (orbital moment $L_z(T_k)$) of the $Cu_3^{R[II]}$ DM complexes with the large Heisenberg $J$ and DM $D_z$ parameters can be observed and used at the higher temperatures, in comparison with the spin chiralities of the $V_3$, $\{Cu_3\}$ NMs with the smaller $D_z$. This shows the advantages of the $Cu_3^{R[II]}$ trimers with the large $D_z$ parameters in the rotating $\hat{H}_r(\phi)$ field over the $V_3$, $\{Cu_3\}$ NMs with small $D_z$ in the possible applications of their chiralities (orbital moment) in molecular devices.

Joint correlated behavior of the net chirality vector $k_{tk}$, the scalar chirality $\chi_{tk}$ and magnetization vector $\hat{M}_{tk}$ of the $Cu_3^{R[II]}$, $V_3^{R[II]}$ trimers in the rotating $\hat{H}_r(\phi)$ field at the temperature $T_k$ is governed by the correlation $\chi_{tk} = 2(k^R_{tk} \cdot \hat{M}_{tk})$. This $k$, $\chi$ behavior demonstrates control (manipulation) of the direction and magnitude of the net vector chirality $k_{tk}$, correlated with the magnitude (sign) of the scalar chirality $\chi_{tk}$ [orbital moment $L_z(T_k)$] of the system by the field rotation and variation of the temperature $T_k$.

Temperature dependence of the vector and scalar spin chiralities, orbital moment and magnetization in the rotating magnetic field, the correlation between the spin chirality and magnetization are of interest for the field control and possible applications of the chiralities of the $Cu_3$, $V_3$ NMs in the molecular based devices and quantum computation.

References


