Present Status of the Israeli FEL: Increasing FEL Power by Electron Beam Energy Boosting

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Abstract
The status of a R&D work aimed on increasing the FEL power by boosting the electron beam energy after the radiation build-up is reported. A fine control of the electron beam energy during the radiation pulse is designed to compensate the small energy degradation during the pulse. Also, a controlled ramp (up or down) in the electron energy during the pulse will be applicable.

Theoretical estimations of the output power in the presence of an electron energy change during the pulse are presented. Two models, showing agreement between them are compared: Analytical model based on the pendulum equation, and, Rigorous 3D FEL interaction model solved numerically.

Another expected result of the design is to further extend the pulse duration with stable conditions and to obtain improved coherency.

The electrical and mechanical lay-outs of the high-voltage boosting (leading to electron beam energy boosting) are also presented.

Introduction
Electrostatic accelerator based FEL’s (EA-FEL) are characterised by long pulse operation. Unlike RF-linac FEL’s having a pulse width limited by the RF part, the EA-FEL’s pulse width is incomparably longer and practically limited by the capability of the power supply to support the system with the required energy. In principle, an ideal power supply can support a CW operation of the FEL. Calculations of a long pulse operation of an FEL show a saturation regime obtained at the end of the energy build-up process, were the output power of the FEL is stable at its maximum. This maximum is related to the FEL characteristics and the operation conditions (such as electron-energy) which are kept constant.

In this paper the possibility of changing the electron energy during the pulse, after the radiation build-up, is considered. Calculations are made in two different methods to estimate the variations in the EA-FEL output power in saturation as a result of change in the electron energy during the pulse. Schematic of a high voltage boost system that supports a change in the electrons energy during the pulse is presented. This system allows a controlled ramp of the electron energy (up or down). This ramp can also be used to compensate the small degradation in the high voltage during the pulse to practically obtain constant electron energy for longer periods. That will lead to improvement of the coherency in the FEL radiation in view of the long pulse operation in constant conditions. In the following sections the two different models are presented and the results are given. Also the designed experimental setup to support the energy variations during the pulse is described.

The FEL 3-D Model
The calculations were carried out in the framework of steady-state, three-dimensional, space-frequency approach, described in more details in [1]. In the approach, the total electromagnetic field oscillated at signal frequency $f_s$ is presented in the frequency domain as an expansion in terms of transverse eigen-modes $e_q(x, y)$ of the cavity, in which the field is excited and propagates:

$$\tilde{E}(r) = \sum_q C_q(z) e_q(x, y) e^{-jkqz}$$

Here the time-domain field is

$$E(r,t) = \text{Re}\left[\tilde{E}(r) e^{+j2\pi f_s t}\right]$$

and $C_q(z)$ is an amplitude coefficient of the mode $q$, which could be found from the excitation equation:

$$\frac{d}{dz} C_q(z) = -\frac{1}{2N_q} e^{+jkqz} \int \int \tilde{J}(r) \cdot e_q^*(x, y) dx dy$$

The power of the electromagnetic field emitted up to the point $z$ can be found as follows:

$$P_{EM}(z) = \sum_q |C_q(z)|^2 \frac{1}{2} \text{Re}\left[ N_q \right]$$

here $N_q$ is the mode normalization power of the mode $q$. 

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As usually accepted we consider the electron beam as consisting of a number of electron clusters or charged quasi-particles, distributed over the beam. Therefore the excitation current can be given in such a form:

\[
\mathbf{J}(\mathbf{r}, t) = -\sum_i q_i \mathbf{v}_i \delta(x-x_i) \delta(y-y_i) \delta(z-z_i(t)) \tag{5}
\]
or in the frequency domain:

\[
\tilde{\mathbf{J}}(\mathbf{r}) = -2\sum_i q_i \mathbf{v}_{zi} \cdot \delta(x-x_i) \delta(y-y_i) e^{-j2\pi f_s t_i(z)} \tag{6}
\]

Substitution of the excitation current (6) into the excitation equation (3) enables one to re-write the last as follows:

\[
\frac{d}{dz} C_q(z) = \frac{1}{N_q} \sum_i \left\{ q_i \mathbf{v}_i \cdot \mathbf{E}_q (x_i,y_i) e^{+j[k_{zq} z - 2\pi f_s t_i(z)]} \right\} \tag{7}
\]

In Eq. (5)-(7) \(q_i, \mathbf{v}_i\) and \(\mathbf{r}_i \equiv \{x_i,y_i,z_i\}\) are the charge, the velocity and the coordinates of the particle with a number \(i\), and

\[
t_i(z) = t_{0i} + \int_0^z \frac{1}{v_{zi}(z')} dz' \tag{8}
\]
is the moment when the particle number \(i\) comes to the point \(z\).

With a known field, a next phase-space position of the particles can be found from the equations of motion:

\[
\frac{d\mathbf{v}_i}{dz} = -\frac{1}{\gamma_i} \left\{ \frac{e}{m v_{zi}} \left[ \mathbf{E} (\mathbf{r}_i, t) + \mathbf{v}_i \times \mathbf{B} (\mathbf{r}_i, t) + \mathbf{v}_i \frac{d\gamma_i}{dz} \right] \right\} \tag{9}
\]

\[
\frac{d\gamma_i}{dz} = -\frac{e}{mc^2 \gamma_i} \mathbf{v}_i \cdot \mathbf{E} (\mathbf{r}_i, t) \tag{10}
\]

Figure 1 demonstrates schematically an FEL operated in oscillator regime. In that case some part of the radiation emitted by the beam is reflected by mirrors and returned to the interaction region, been forced to interact with a new portion of the driving current. Then the total electromagnetic field, emitted after \(N\) round-trips of the radiation in the resonator, may be found by:

\[
\tilde{\mathbf{E}}_{tot}(\mathbf{r}) = \sum_q C_q^{tot}(z) \mathbf{E}_q(x,y) e^{-j k_{zq} z} \tag{11}
\]

Here the expansion coefficient of the total field is

\[
C_q^{tot}(z) = \sum_{n=1}^N C_q^{(n)}(z) \tag{12}
\]

and the coefficients of the field emitted after \(n\) round trips of the radiation are defined by the recursion relation:

\[
C_q^{(n+1)}(z = 0) = \Gamma C_q^{(n)}(z = L_w) e^{-j k_{zq} L_w} \tag{13}
\]

The above equations form a closed set of non-linear equations, which enables one to calculate the both radiated field and the beam trajectory. The model was realized in FEL3D numerical code and applied for the calculations considered in the next section.

THE "PARAMETRIC" PENDULUM EQUATION MODEL

The oscillation build-up and saturation process of the FEL oscillator were analyzed numerically in Ref. [3,4]. At saturation the radiation field inside the cavity is built up, and the small signal assumptions are not valid. The electron dynamics is described in the combined wiggler and radiation wave fields (the ponderomotive wave) in terms of the pendulum model [5]:

\[
\frac{d^2\Psi}{dz^2} = -K_s^2 \sin \Psi \tag{14}
\]

\[
\theta = \frac{d\Psi}{dz} \tag{15}
\]

where \(\Psi(z)\) is the phase of the electron relative to the ponderomotive wave: \(\Psi(z) = \int_0^z \frac{d\theta(\omega, z')}{v_z(z')} + \Psi_0\), and the detuning parameter is \(\theta(\omega, z) = \frac{\omega}{v_z(z)} - k_{zq}(\omega) - k_w\).

\(k_{zq}(\omega)\) is the axial wavenumber of mode \(q\), and \(k_w = 2\pi/\lambda_w\) is the wiggler wavenumber. The Synchrotron oscillation wavenumber \(K_s\) is given by
where \( E \) and \( P \) are the circulating radiation field and power in the saturating oscillator cavity (we assume high round-trip resonator reflectivity, and therefore constant power \(-P\) along the resonator).

As well known, the pendulum equation (14) can be integrated once, resulting in a picture of open and closed trajectories in \( \theta - \Psi \) phase-space (Fig. 2). This picture can also be viewed as a display of electron energy vs. phase trajectories, if one uses the differential linear relation between \( \gamma \) and \( \theta \) near the synchronism energy \( \gamma_0 \) (Fig. 3).

\[
A_p = \frac{1}{2\pi^2} \cdot \frac{(\gamma_0 \gamma_0' \beta_0^2)}{a_w^2} \cdot \frac{A_{\text{em}} \lambda^2}{L^4} \cdot \frac{(mc^2/e)^2}{Z_q}
\]

\[
P = A_p K_s^4
\]

Figure 2: Phase-Space Trajectories of the Pendulum Equation.

**OSCILLATION BUILD-UP EA-FEL**

The oscillation build-up dynamics as a result of the two models are presented in Fig. 4. all the calculations are made for a single frequency. The result for a constant electron energy (1.42 MeV) is shown in green. This calculation is done by the pendulum equation model. For the same frequency, a case with a step in the electron energy is calculated (red curve). The initial voltage is 1.4 MV and saturation is obtained. Than a voltage step is applied to 1.42 MV. As a result a new saturation level is obtained which is higher than the former level of a stable voltage. Thus, for the same final (i.e 1.42 MV) conditions two saturation levels are possible.

A verification of the red curve behaviour is obtained by the FEL 3D model (violet curve). Although different levels of saturation are obtained, the behavior is similar, where a jump in the saturation level is obtained. The differences between the models can be related to the differences in the assumptions in the models.

![Figure 3: Dynamics of electron beam trapping and synchrotron oscillation at steady state saturation stage of an FEL oscillator.](image)

**ENERGY BOOST SYSTEM**

Since the EA-FEL is capable of a long pulse operation, a coherent operation in high power is applicable [6]. Still. A practical experimental difficulty is the electrons the hit the terminal parts and reduce the voltage. These electrons can be related to non-ideal transport conditions and to returning electrons that could not reach the collector. This phenomena cause a drift in radiation frequency that is related to the drift in the voltage drop. Correcting this voltage drop was one of the motivations to the work presented here. But, if a correction system is designed, it can be used to more than a voltage correction but also to voltage control in a desired manner such as a ramp.

The voltage ramp control system is presented in Fig. 5. it is placed in the high voltage terminal and its output is added to the terminal voltage. A capacitor is charged to voltage of up to 30 kV. During the pulse it is partly discharge through a selected resistor to the terminal. Therefore the voltage of the terminal is raised. By selecting the resistor and voltage the rate of the voltage raise can be determined. Therefore a compensation to the electrons hitting the terminal can be achieved in order to stabilize the voltage conditions and as a result the frequency as well.

Also a ramp (positive or negative) in the voltage can be applied in order to enhance the saturation level as predicted by the theory.
CONCLUSION

The possibility to achieve higher extraction efficiency and therefore higher operation power of the EA-FEL by electron energy step is theoretically possible. Bi-stable saturation conditions are obtained for different initial conditions. An experimental setup is under construction in order to demonstrate the effect.

REFERENCES


