Modeling Of Ground Motions For Tbilisi Region With Site Effects

Paata Rekvava[1], Ketevan Mdivani[1]

[1]Georgian National Association for Engineering Seismology and Earthquake Engineering,
Tbilisi, Georgia
rekvavapaata@yahoo.com, mdivanik@yahoo.com

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ABSTRACT

The utilization of time histories of earthquake ground motion has grown considerably in the field of earthquake engineering. It is very unlikely, however, that recordings of earthquake ground motion will be available for all sites and conditions of interest. Hence, there is a need for efficient methods for the simulation of strong ground motion for a given region.

Due to lack of the real strong ground motion records the objective of this research is to develop a methodology for rapid generation of horizontal and vertical components of earthquake ground motion at any site for Tbilisi region (within 50 km). The model developed in this study provides simulation of ground motion over a wide range of magnitudes and distances at 8 earthquake source s zones of Tbilisi region.

The research includes three main topics: (i) the stochastic simulation of earthquake ground motion at a given site of the city of Tbilisi; (ii) the estimation of acceleration time histories at a given site using the direct method of engineering seismology considering soil conditions based on the theory of the reflected waves and (iii) calculation of horizontal and vertical acceleration response spectrum for main sites of Tbilisi territory.

The obtained results in the terms of acceleration response spectra and spectral dynamic coefficients can be widely applied in the practice of earthquake engineering in Georgia.

Introduction

In the practice of earthquake engineering an earthquake effect quantitatively is classified according to the seismic scale and by the building code. For this purpose are used Peak Ground Acceleration (PGA), Peak Ground Velocity (PGV) and Peak Ground Displacement (PGD) in the seismic scale.

In the building code the seismic action usually is represented by an elastic acceleration response spectrum and the acceleration time-histories.

It should be noted that, each earthquake represents individual process, which is generated under certain geographic and geological conditions, its destructive effect first of all depends on the seismic source magnitude and the epicentral distance.

The elastic acceleration response spectrum shape depends on the earthquake generation mechanism and ground response in the site of interest. Therefore, the elastic acceleration response spectra defined according to the recorded accelerograms in different regions, differ from each other and reflect only local site conditions.

The time-histories dynamic analysis provides the evaluation of seismic demand of structures using the recorded and artificial or simulated accelerograms that gives information on earthquake intensity, its frequency and duration, i.e. it does not exclude time factor as it occurs in the response spectrum analysis.

Proceeding from the regulations on seismic action basic conception given in [1], selection of the elastic response spectrum shape in the country or part of the country is possible from the certain country National annexes that are worked out by local Authorities. In accordance with the recommendations suggested by Eurocode 8 [1] deep geological data of the construction site should be considered and the horizontal and vertical elastic response spectra should be computed taking into account the seismic sources and the earthquake magnitudes generated from them.

In general, lack of the strong earthquake records statistic package creates certain problems for the elastic response spectra and the dynamic coefficient spectral curves elaboration. It should be noted that, an example of such problems is Tbilisi, the capital of Georgia.

For Tbilisi region (within 50 km) records of the strong earthquakes data are limited. During last 100 years at the territory of Tbilisi city about hundred weak earthquakes took place. Local strong earthquake occurred only on April 25, 2002, under the central part on the city with magnitude M=4.5 and recorded on the bedrock peak horizontal acceleration of 0,11g, which was amplified to the range of 0.20 to 0.30g due to dynamic response of surface soil deposits.

It is evident that, on the basis of the weak and rare earthquakes real records formation of the seismic action specified regional model is impossible. In such conditions the most straightforward procedure is to generate ground motion time-histories using of regional earthquake sources zones parameters and classification according to the soils seismological and geological properties available on the territory of Tbilisi.

At the same time, according to the Georgian Building Code [2] Tbilisi is located in the seismic zone of intensity 8 degree by the MSK-64, with a maximum horizontal acceleration equals 0.17g and a return period of earthquakes 2500 years (2%/in 50 years). The spectral dynamic coefficient is determined for grounds of hard (I), medium (II) and soft (III) categories and without special investigations its maximum value for all three categories grounds equals 2.5.

It should be noted, that from the earthquake source zones of Tbilisi region at the territory of the city there are expected the earthquakes with magnitudes M=5.0_7.0 and corresponding seismic generated kinematics of shifting as reverse and dextral strike slip [3]. Therefore it is necessary to define more precisely the spectral curves of dynamic coefficient considering seismological and geological properties of the grounds within the city and adjacent area. Under such conditions it is important to simulate regional seismic action in the form of the acceleration of time-histories and to elaborate the elastic response spectra and the three-component spectral curves of dynamic coefficient for various sites of Tbilisi.

There are several models in the literature for numerical simulation of earthquake ground motion. The ground motion simulation models can be classified into two categories: geophysical models and engineering models, which estimate the ground motion in fundamentally different ways [3,4].

The objective of this study is to develop a methodology for simulation ground motions and evaluation the acceleration response spectra and the three components spectral curves of dynamic coefficient at any site for Tbilisi territory considering the regional seismological characteristics and geological conditions for the site of interest.

The proposed approach includes three main topics: (i) the stochastic simulation of earthquake ground motion at a given site of the city of Tbilisi; (ii) estimation of acceleration time histories at a given site using the direct method of engineering seismology taking into account a soil properties based on the theory of the multiple reflected waves (iii) calculation of the horizontal and vertical elastic response spectra and corresponding the spectral curves of dynamic coefficient for main sites of Tbilisi territory.

Stochastic Simulation of Earthquake Ground Motion

For simulation of possible seismic ground moutions on the territory of Tbilisi city in this paper is employed the discrete nonstationary Gaussian stochastic process represented as [5]

$$A_{gi}(t) = E_i(t)\ X_i(t), \quad (i=1,2,3) \qquad (1)$$

where $A_{gi}(t)$ determines of ground accelerationin in the direction of three principal orthogonal axses with zero cross correlation between of components; $E_i(t)$ is the deterministic normalized envelope function or modulating function; $X_i(t)$ represents a typical realization of the stationary filtered white-noise process.

Normalized stationary random function with zero mean and unit-variance is characterized by $K(\tau)$ function of correlation as

$$K(\tau)=e^{-\alpha_j|\tau|}(\cos\omega_j\tau+\alpha_j/\omega_j\sin\omega_j|\tau|) \qquad (2)$$

where α is correlation coefficient characterizing bandwidth of the process ; ω is circular process frequency; j represents a ordinal number of process .

The modulating function $E_i(t)$ is defined in terms of so-called Berlag impulse and with $|E_i(t)|_{max}=1$ is given by

$$E_i(t) = \varepsilon\ t \exp(1-\varepsilon t) \qquad (3)$$

where ε controls the shape of the envelope function and determines the effective duration and process nonstationarity.

Generalizing the form in Eq.1, the horizontal and vertical components of the process can be written as

$$\begin{aligned} A_{g1}(t) &= \kappa\sigma_1\varepsilon t\exp(1-\varepsilon t)x_1(t) \\ A_{g2}(t) &= \eta\sigma_2\varepsilon t\exp(1-\varepsilon t)x_2(t) \\ A_{g3}(t) &= \upsilon\sigma_3\varepsilon t\exp(1-\varepsilon t)x_3(t) \end{aligned} \qquad (4)$$

where σ_i is a mean square value of acceleration in the direction of principal axses and denotes random process intensity that is defined by its variance; k,η da υ are corrective factors of the value of the horizontal and vertical components which are accordingly equal to 1.0, 0.85 and 0.7.

Thus, the formulation in Eq.4 is completely determined with fixed values of dominant frequency ω_j using three parameters: α, ε and σ which are depended on regional seismological and geological conditions or in the simple form on the earthquake magnitude, hypocentral distance, dominant frequency and ground characteristics at the site.

On the basis of proposed stochastic ground motion model formulated in Eq.4 the software package ACCSIM [3] was developed, which allows to generate the multiple artificial accelerograms of the predicted earthquakes.

Parameters Estimation

For estimation of the maximum macroseismic intensity ITb of the expected earthquake on the territory of Tbilisi city from the earthquake sources zones of Tbilisi region the two various expressions are applied [6]:

for small earthquakes (M_s<6)

$$I_{Tb}=1.5M_s–3.4\lg R+3.1 \qquad (5)$$

for strong earthquakes (M_s>=6)

$$I_{Tb}=1.5M_s–4.7\lg R+4.0 \qquad (6)$$

where M_s is surface-wave magnitude; $R= (\Delta^2+h^2)^{1/2}$ is hypocentral distance; Δ is epicentral distance; h-focal depth.

The resulting equation for larger horizontal values of peak horizontal acceleration is defined by [7]

$$\log PGA_{h1}=0.72+0.44M_s-\log R-0.00231K+0.28p \qquad (7)$$

and

$$K = \sqrt{\Delta^2 + h^2 + 4.5^2} \qquad (8)$$

where p is 0 for 50-percentile values and 1for 84-percentile.

For the determination of dominant period T of ground motion empirical relations between the surface-wave magnitude of the earthquake and a hypocentral distance derived for shallow-focus near-source earthquakes under an average soil site conditions are calculated by the following formula [8]

$$\lg T=0.15M_s+0.25\lg R+C_1+C_2\pm0.2 \qquad (9)$$

where C_1 is parameter of fault mechanism (C_1=-0.1 for reverse, C_1=0 for strike slip, C_1=0.1- for dextral strike slip); C_2–coefficient of influence not taken into consideration factors that is equal to 1.11;

Duration of the intesive phase of ground motion is computed by

$$\lg D=0.15M_s+0.50\lg R+C_1+C_2+C_3\pm0.30 \qquad (10)$$

where C_1 – is parameter of fault mechanism (C_1=-0.25 for reverse, C_1=0.0 for strike slip, C_1=-0.12 for dextral strike slip; C_2=-0.15 for hard ground; C_2=0 for medium ground; C_2=0.15 for soft ground; a mean value of ratio C_3 is equal to 1.3.

The calculated parameters for the borderline territory of Tbilisi city are listed in Table 1. It should be noted, that for computation of PGA has been used 84-percentile. On the basis of empirical data the more intensive horizontal component of PGA_{h1} is obtained 1.28 times greater than other one and the vertical component is 2/3 of the maximum horizontal component.

For 10 sites of Tbilisi city territory (350 square kilometers, Fig.1) were also determined minimum hypocentral distances, PGA for 2% and 1% probabilities of being exceeded in 50 years, values of the dominant periods and duration of oscillation. As an illustration in Table 2 are given parameters generated from the high potential seismic generating zone #3 which is situated to the north-west of Tbilisi region. Note, that values of PGA in Table 2 correspond to 2% probability of being exceeded in 50 years and are by 15% less than computed for 1% probability of being exceeded in 50 years.

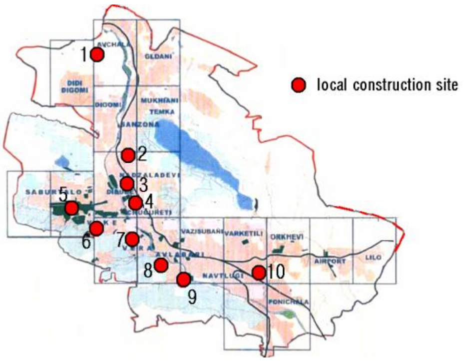

Fig. 1. Location of the sites on the territory of the city

Table 1. Quantitative Characteristics of the Predicted Ground Motion on the Borderline of Tbilisi City

Zone #	R (km)	I_{TB} (deg)	T (sec)	D (sec)	PGA_{h1} (m/sec^2)	PGA_{h2} (m/sec^2)	PGA_{h3} (m/sec^2)
From focus with M=5.0							
2	10.6	7	0.13	1.63	2.11	1.65	1.41
5	8.38	7	0.12	1.45	2.20	1.72	1.47
From focus with M=5.5							
7	10.7	8	0.15	2.06	2.38	1.86	1.59
From focus with M=6.0							
4	11.2	8	0.18	2.66	2.53	1.98	1.69
6	10.0	8	0.18	2.51	2.57	2.01	1.72
8	16.0	7	0.2	3.18	2.38	1.86	1.67
From focus with M=6.5							
1	29.4	7	0.28	5.42	2.32	1.81	1.55
From focus with M=7.0							
3	16.3	9	0.28	5.08	2.81	2.20	1.88

Table 2. Quantitative Characteristics of the Predicted Ground Motion for the Concrete Sites of Tbilisi City

Zone #	M	Parameters	Site #									
			1	2	3	4	5	6	7	8	9	10
3	7.0	R_{min}, km	15.6	20.0	21.2	23.3	18.1	21.6	25.1	26.8	30.0	33.7
		PGA_{h1}, m/sec^2	2.83	2.72	2.69	2.65	2.77	2.69	2.62	2.58	2.53	2.47
		PGA_{h2}, m/sec^2	2.21	2.13	2.10	2.07	2.16	2.10	2.04	2.02	1.98	1.93
		PGA_{h3},m/sec^2	1.89	1.81	1.80	1.77	1.85	1.79	1.74	1.72	1.69	1.65
		T, sec	0.28	0.30	0.30	0.31	0.29	0.305	0.32	0.32	0.33	0.34
		D, sec	4.98	5.63	5.8	6.08	5.35	5.86	6.3	6.52	6.9	7.31

The main parameter ω_j of the ground motion model has been determined based on the Eq.9 using the expression:

$$\omega_j = 2\pi / T_j \qquad (11)$$

The value of the correlation degree characterizing parameter α was evaluated based on the analysis of the earthquakes records data [3] depending on ω and for 1(x), 2(y) da 3(z) components consists of

$$\alpha_{j1}=0.204\omega_j; \quad \alpha_{j2}=0.253\omega_j; \quad \alpha_{j3}=0.41\omega_j; \qquad (12)$$

The mean square value of acceleration σ was accepted considering that

$$\sigma_i=PGA_i/3, \quad i=1,2,3 \qquad (13)$$

The parameter ε is determined on the basis of the given duration of intensive oscillations above-mentioned records and is equal to

$$\varepsilon_j = 0.02\omega_j \qquad (14)$$

Thus calculated parameters are represented in Table 3, but mean square values of the horizontal and vertical accelerations for earthquake generated from the high potential seismic generating zone #3 are given in Table 4.

Table 3. Parameters for Genaration of Regional Synthetic Accelerograms

Zone 1	ω_j, sec^{-1}	α_{j1}, sec^{-1}	α_{j2}, sec^{-1}	α_{j3}, sec^{-1}	ε_j, sec^{-1}	$dt=0.04T_j$, sec
From focus with M=5						
2	48.3	9.85	12.1	19.8	0.97	0.005
5	52.3	10.67	13.1	21.4	1.05	0.0048
From focus with M=5.5						
7	41.8	8.53	10.5	17.1	0.84	0.006
From focus with M=6						
4	34.8	7.1	8.7	14.3	0.7	0.007
6	34.8	7.1	8.7	14.3	0.7	0.007
8	31.4	6.4	7.9	12.9	0.63	0.008
From focus with M=6.5						
1	22.4	4.5	5.6	9.2	0.45	0.011
From focus with M=7						
3	22.4	4.6	5.6	9.2	0.45	0.011

Table 4. Mean Square Values of Aaccelerations for Concrete Sites

Mean square value of acceleration m/sec^2	Probability of exceeding in 50 years	Site #									
		1	2	3	4	5	6	7	8	9	10
σ_1	2% /50	94	91	90	88	92	90	87	84	84	82
	1% /50	109	104	103	102	106	103	100	97	97	95
σ_2	2% /50	73	71	70	69	72	70	68	67	66	64
	1% /50	85	81	81	79	83	80	78	77	76	74
σ_3	2% /50	63	60	60	59	62	60	58	57	56	55

	1% /50	72	70	69	68	71	69	67	66	65	63

Determination of Multilayer Ground Motion Based on the Theory of Multiple Reflected Waves

Method of the multiple reflected waves gives a possibility to determine for a concrete territory by geologic profile conformity to natural laws of seismic oscillation of the multilayer ground surface, under motion of rockbed as foundation according a law of given accelerogram.

For analytical drawing of accelerogram of the ground free surface oscillation let consider, a wave picture at any time in the ground area, with different thickness and horizontal borderline. It is assumed that the ground is elastic and waves are propagated in the vertical direction (Fig. 2). In the form of seismic influence in this case is used recorded on the rockbed accelerogram from the database of ground motions with known earthquake [3].

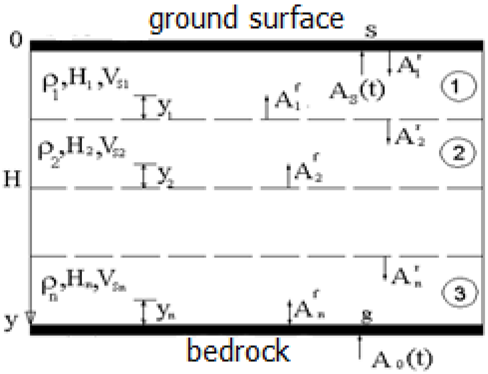

Fig. 2. Design model of nonhomogeneous ground

In the Fig. 2 are accepted following designations: $A_i^f(t)$ is value of acceleration's wave function at the t time on the bottom level of the i-th layer; $A_i^r(t)$ is value of acceleration's wave function at the *t* time on the top level of the i-th layer; $A_0(t)$ is accelerogram on the level of hard rock.

For any i-th layer of ground the wave equation of shear oscillations can be written as [9]:

$$\frac{\partial^2 A_i(t)}{\partial t^2} - V_{si}^2 \frac{\partial^2 A_i(t)}{\partial y^2} = 0 \tag{15}$$

where A_i(t) is the acceleration of ground layer particles; t is the time; y represents the coordinate of ground particles in the vertical direction; V_s is the velocity of the shear wave propagation.

Solution of the equation (4.1) is given by

$$A_i^r(t) = \alpha_{i-1,i} A_{i-1}^r(t - \tau_{i-1}) + \beta_{i,i-1} A_i^f(t - \tau_i) \tag{16}$$

where $\alpha_{i,i-1}$ is the factor of refraction under passing of wave from i-1-th to i-th layer; $\beta_{i,i-1}$ the factor of wave reflection on the borderline between i and i-1 layers; τ_i represents the time of wave passage in the i-th layer ($\tau_i = H_i / V_{si}$, where H_i and V_{si} are accordingly the thickness of ground layer and the velocity of the shear wave propagation in the i-th layer).

α and β factors are defined by

$$\alpha_{i-1,1} = 2\rho_{i-1} V_{s,i-1} / \left(\rho_{i-1} V_{s,i-1} + \rho_i V_{si}\right) \tag{17}$$

$$\beta_{i,i-1} = \left(V_{si}\rho_i - \rho_{i-1} V_{s,i-1}\right) / \left(V_{si}\rho_i + V_{s,i-1}\rho_{i-1}\right) \tag{18}$$

where ρ_i is a density of i-th ground layer.

Hence, finally solution of the direct problem of engineering seismology can be represented by the recurrent relations as [9]:

$$\begin{aligned}
A_1^r(t) &= A_1^f(t-\tau_1),\\
A_1^f(t) &= \alpha_{2,1} A_2^f(t-\tau_2) + \beta_{1,2} A_1^r(t-\tau_1),\\
A_2^r(t) &= \alpha_{2,1} A_1^r(t-\tau_1) + \beta_{2,1} A_2^f(t-\tau_2),\\
A_2^f(t) &= \alpha_{3,2} A_3^f(t-\tau_3) + \beta_{2,3} A_2^r(t-\tau_2),\\
A_i^r(t) &= \alpha_{i-1,i} A_{i-1}^r(t-\tau_{i-1}) + \beta_{i,i-1} A_i^f(t-\tau_i),\\
A_i^f(t) &= \alpha_{i+1,i} A_{i+1}^r(t-\tau_{i+1}) + \beta_{i,i+1} A_i^r(t-\tau_i),\\
A_n^r(t) &= \alpha_{n-1,n} A_{n-1}^r(t-\tau_{n-1}) + \beta_{n,n-1} A_n^f(t-\tau_n),\\
A_n^f(t) &= \alpha_{n+1,n} A_0(t) + \beta_{n,n+1} A_n^r(t-\tau_n).
\end{aligned} \tag{19}$$

Oscillation of the particles from the bottom of i-th layer on the level of y_i can be calculated according to

$$A_i^{y_i}(t) = A_i^f\left(t - y_i / V_{si}\right) + A_i^r\left(t - \left(H_i - y_i\right)/V_{si}\right) \tag{20}$$

Thus, the developed algorithm of solution the direct problem of engineering seismology is realized by the software package GAFART [3].

Simulation Results for Tbilisi City Sites

The software ACCIM was used for generation of the horizontal and vertical components of synthetic accelerograms corresponding possible seismic source zones of Tbilisi region, given in Table 1. Discrete step of the simulated accelerograms was taken equal to 0.04T. When assessing the probabilistic mean elastic response spectra and the dynamic coefficient spectral curves for all sites, which are presented in Fig. 1, the required number of realizations was reduced for each synthetic accelerogram up to 20 realizations. The most novel aspect of this extension is elaboration of proper three components dynamic coefficient spectral curves, which are computed for 50 years exposure time and 2% and 1% probabilities of exceeding.

As an example, Fig. 3 shows pairs of the probabilistic mean elastic acceleration response spectra for 5% damping and the corresponding spectral curve of dynamic coefficient of synthetic motion, generated from the source zone ^{1}3, for site ^{1}8 of soft soil characteristics for 2% in 50 years.

Using the software GAFART an influence of a typical earthquake and local geological conditions upon forming the elastic acceleration response spectra for the abovementioned sites was studied. With that and in view from the data set [3] was selected five recorded on the bedrock accelerograms (EL Centro-1940, M = 6.7, Santa Barbara-1980, Montenegro-1979, M = 7.0, Friuli 1976, M = 6.0, and Tbilisi-2002, M = 4.5), which are different from each other by parameters of PGA, dominant period (T) and duration (D), but by the magnitude and epicentral distance are close to predictable earthquakes characteristics for Tbilisi region.

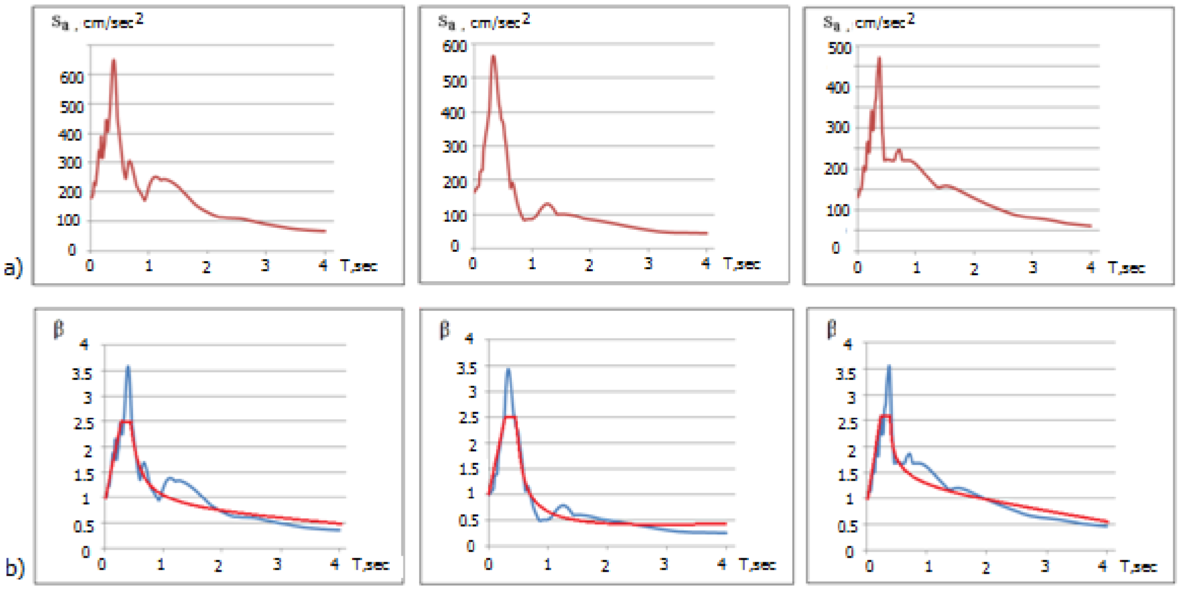

Fig. 3. Generated from zone #3 for site #8 x,y,z components of acceleration response spectra (a) and spectral curves of dynamic coefficient

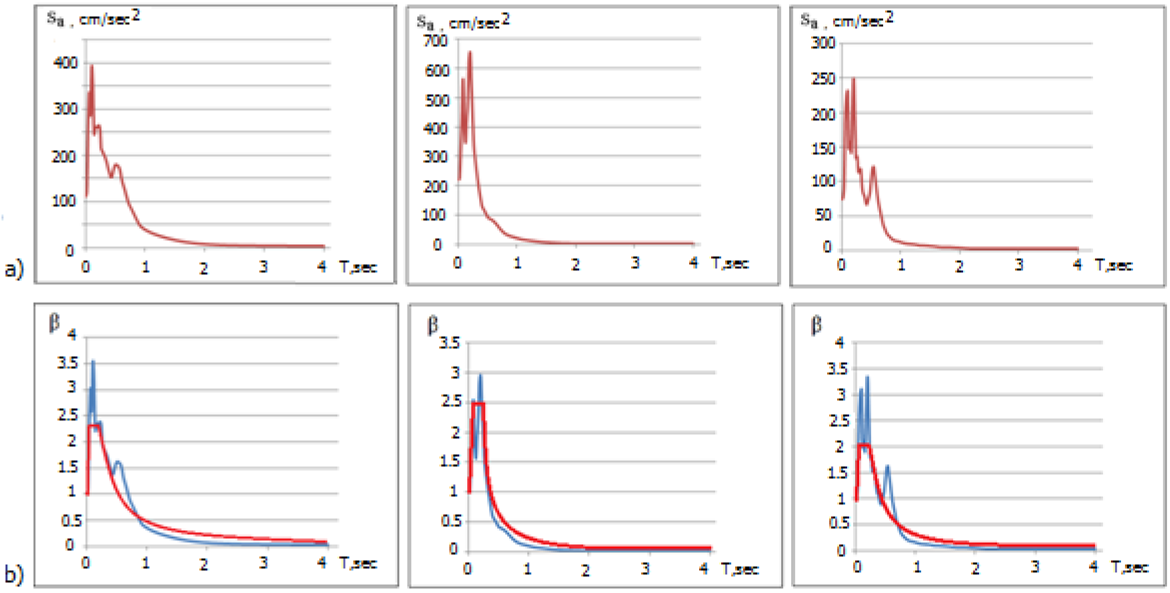

Fig. 4. Three components acceleration response spectra (a) and spectral curves of dynamic coefficient (b) for site #8 resulting from the accelerogram calculated at the free surface from recording "Tbilisi- 2002" of the depth z=-43.1m

Considering the soil profile properties (thickness, density, shear wave velocity) of these sites, received from geological test, on the basis of the abovementioned recorded ground motions were calculated the three-components of time-histories on the ground surface of the sites. Then at the final phase of analysis the three components of elastic acceleration response

spectra and spectral curves of dynamic coefficient have been plotted. Fig.4 displays the effect of local soil condition on the dynamic coefficient for the concrete site ^{1}8 under action of Tbilisi-2002 records.
According to the obtained results the value of the amplification factor as ratio of the maximum accelerations on the ground surface and on the bedrock was computed. The analysis shows that in the given soil properties of the sites under investigation the amplification factors are changed from 1.5 to 2.6.

Summary

The complex approach of simulation ground motion and construction of the horizontal and vertical elastic response spectra and corresponding dynamic coefficient spectral curves are proposed, which account for the location of the earthquake source zones and seismological and geological characteristics of the Tbilisi region and concrete construction sites.

Based on the empirical relations and characteristics of the earthquake source zones the values of mean accelerations of ground motion for 2%/ and 1% probabilities of being exceeded in 50 years expected in the sites of Tbilisi city has been determined and appropriate probabilistic horizontal and vertical elastic response spectra and spectral dynamic coefficients are calculated (can be used in seismic design and analysis of structures).

On the territory under examination for the concrete construction sites in result of experimental research the dynamic parameters of soil geological layers are determined and based on the theory of multiple reflected waves the ground motion for the sites of Tbilisi city horizontal and vertical elastic response spectra and spectral dynamic coefficients are calculated, their corrected shapes considering the local sites conditions for II and III category of soil are constructed, which can be widely applied in the practice of earthquake engineering in Georgia.

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