CONTROL IMPROVEMENT OF UNDER-DAMPED SYSTEMS
AND STRUCTURES BY INPUT SHAPING

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\textbf{Abstract} - Vibrations exist all around us. Many examples of such systems range from the positioning of disk drives head to cranes and large space structures spacecrafts, robots, etc. It is a serious problem in mechanical systems that are required to perform precise motion in the presence of structural flexibility. This paper investigates the control of flexible systems using Input Shaping (IS) technique. It specifies an input function that will drive the system and performs point to point movement with minimal oscillation or without vibration at all. The “Two Points and Command” method is proposed to analysis of lightly damped system and to adaptive calculation of the input shaper coefficients. The updated techniques of the adaptation are proposed. The experimental setup is pendulum system. It well imitates lightly damped systems that will continue to oscillate long after the desired move has ended. It considers only both analyzing plant and input command need to reduce vibrations. The results clearly proved the suitability of the IS algorithm. Simulation of the output response and desired shaped command is done using MATLAB.

\textbf{Keywords:} vibration, input shaping, lightly damped systems, adaptive control

1. \textbf{INTRODUCTION}

Design of optimal control with constraints on base command has been an interesting issue for decades. Historically, input shaping dates from the late 1950’s. Originally named “Posicast Control”, the initial development of input shaping is largely credited to Smith (1 and 2). However, there seems to have been one notable precursor. In the early 1950’s, Calvert and Gimpel developed a time-delay based vibration filter named “Signal Component Control” (3). But their solution did not contain the convenient closed-form description offered by Smith. Since this initial work, there have been many developments in the area of input shaping control. Early forms of command generators, such as posicast control, suffered from poor robustness properties. The recently proposed robust command generators have proven very beneficial for real systems. Swigert (4) proposed techniques for the determination of torque profiles which considered the sensitivity of the terminal states to variations in the model parameters. Other examples, input shapers have been developed that are robust to natural frequency modeling errors, the first of which was called the Zero Vibration and Derivative (ZVD) shaper (5). Input shapers for multi-mode systems have also been designed (5, 6 and 7). Adaptive shaper techniques have been proposed by several authors (8, 9 and 10) etc.
2. INPUT SHAPING REVIEW

2.1 Single Mode Shaping

When a system has just one identical vibration as response to step command, need to reduce it by Single Mode Shaping. An early form of input shaper was the use of Posicast control. This control splits the reference signal into two parts. The size of the steps and the delay before introducing the second step are derived from the system dynamics. The modern technology that we call impulse shaping is a few generations removed from posicast control idea. The original work is done in this area by Singer (11).

Input Shaping is a command generation technique which attempts to impart zero energy into a system at the frequencies at which it will vibrate. In order to ensure zero energy at the vibration frequencies, the commands given to the system must be modified, thus the term Input Shaping. Once the correct command for the system is found, the result will be a system that has no energy at frequencies for which it will vibrate, thus no vibration.

![Diagram of input shaping process](image)

**Figure 1:** Input shaping process

The process relates back to the use of a series of impulses, which will cause zero vibration in the system; this series of impulses or shaper when convolved with the original command to the system yields a response that also causes zero vibration. The convolution process is shown in Fig.1.

A Zero Vibration, ZV, input shaper is the simplest input shaper. The only constraints are minimal time and zero vibration at the modeling frequency. If these constraints are satisfied, the ZV shaper has the form of impulse amplitudes $A_i$ and times $t_i$. The ZV shaper is useful in situations where the parameters of the system are known with a high level of accuracy. Also, if little faith is held in the input shaping approach, the application will never increase vibration beyond the level before shaping.

A Zero Vibration and Derivative, ZVD, shaper is a command generation scheme designed to make the input-shaping process more robust to modeling error. If another constraint is added to the formulation of the shaper by setting the derivative of the vibration with respect to frequency equal to zero.

The application of ZVD shapers is for systems where rise time is still important, but either the system will change with time or the model is not accurate. If the model’s inaccuracy cannot be controlled with a ZVD shaper then other shaping techniques are available, such as those described in the subsequent sections.

It is possible to generate a more robust shaper by forming the second derivative of the residual vibration equation and setting it equal to zero. The shaper that results from satisfying this additional constraint is called a ZVDD shaper. This additional constraint
increases the robustness, but also increases the shaper duration by one half period of the vibration.

ZVDD shaper consists of four evenly spaced impulses lasting 1.5 periods of vibration. ZVDD is three ZV shapers convolved together. The advantage to doing this is that the input shaper parameters have less freedom, thereby simplifying the solution routine. However, by restricting the choice of input shaper parameters, the solution space is also restricted, meaning that there is the potential for optimal solutions to be missed.

The matrix form of the three main Zero Vibration Shapers is shown below.

Table 1: Basic Shaper Coefficients

<table>
<thead>
<tr>
<th></th>
<th>A_j</th>
<th>t_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZV</td>
<td>[ \frac{1}{1+K} ]</td>
<td>[ \frac{K}{1+K} ]</td>
</tr>
<tr>
<td>ZVD</td>
<td>[ \frac{2K}{1+2K+K^2} ]</td>
<td>[ \frac{K^2}{1+2K+K^2} ]</td>
</tr>
<tr>
<td>ZVDD</td>
<td>[ \frac{1}{0.5T_d} ]</td>
<td>[ \frac{K^2}{T} ]</td>
</tr>
</tbody>
</table>

The graphical presentation of Undamped systems ( \( K = e^{-\frac{\zeta \alpha}{\sqrt{1-\zeta^2}}} = 1 \); \( \zeta = 0 \) ) is on Fig.2.

Figure 2: Three main zero vibration shapers

2.2 Multimode Shaping

In systems with more than one mode of vibration, two schemes can be used to generate commands for the system. The first is by simply convolving together multiple Input Shapers designed for each specific mode. The result of this process is a longer input shaper, which can deal with each mode specifically.

Another way to design the shaper is to solve the constraint equations for the two modes simultaneously. This method results in vibration reduction near the modeling frequencies, but does not yield as much suppression of the high modes. However, a simultaneous
shaper is never longer than a convolved shaper, and it is often significantly shorter. This advantage in speed can be important for slow oscillations.

A comparison of the sensitivity curves for the direct solution and the convolved solution is shown in Fig. 3.

**Figure 3:** Convolved and direct two-mode ZVD-ZVD shapers for 1 Hz and 2.5 Hz

Convolution is simple to calculate, easy to program and implement, and maintains high robustness. Direct solution produces a shorter sequence, but is more complex in calculation, harder to implement, and loses some robustness. Different applications will call for different methods. When reducing the time lag in the move is critical, direct solution is the best choice. An application with high modes which tend to move around a lot will call for convolution. And in many cases, a combination of the two is ideal. A list of the qualities of both methods is given in table 2.

**Table 2: Comparison of Multimode Shapers**

<table>
<thead>
<tr>
<th></th>
<th>Convolution Method</th>
<th>Direct Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of impulses</td>
<td>$\prod_{i=1}^{m} n_i$</td>
<td>$\sum_{i=1}^{m} (n_i-1) + 1$</td>
</tr>
<tr>
<td>Length of sequence</td>
<td>$\sum_{i=1}^{m} l_i$</td>
<td>$?&lt; \sum_{i=1}^{m} l_i$</td>
</tr>
<tr>
<td>Calculation complexity</td>
<td>Simple</td>
<td>Complex</td>
</tr>
<tr>
<td>Robustness</td>
<td>Very robust</td>
<td>Less robust</td>
</tr>
</tbody>
</table>

- ‘m’ is the number of modes, ‘n’ is the number of impulses per mode.
- ‘l’ is the length of each sequence.
3. SYSTEM COEFFICIENTS

The theoretical transfer function of the system with vibrations should be like:

\[ G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} \]

When system frequency (\( \omega \)) and damping ratio (\( \zeta \)) are known, it can depict the system. If there is no way to get the analytic solution, it's possible just to record the data. On the figure 4 is the response of the vibration system. It's not hard to measure the Peaks (Amplitude and Time values). The peaks get the frequency of the system:

\[ T = \frac{2\pi}{\omega} \approx t_{\text{peak2}} - t_{\text{peak1}}(\omega) \]

![Figure 4: The step response reaction and extreme points](image)

Then \( \omega = \frac{2\pi}{(t_{\text{peak2}} - t_{\text{peak1}})} \) and frequency depends on two continuous positive or negatives extremes. In this case the command is known and just two sequenced points ('a' and 'b' or 'a' and 'c') help to find the damping ratio (Table 3).

**Table 3: Two Points and Command method**

<table>
<thead>
<tr>
<th>Extreme points</th>
<th>'a' and 'b'</th>
<th>'a' and 'c'</th>
</tr>
</thead>
<tbody>
<tr>
<td>The proportion between the points</td>
<td>( \frac{a}{b} = e^{\pi \zeta} )</td>
<td>( \frac{a}{c} = e^{2\pi \zeta} )</td>
</tr>
<tr>
<td>Damping ratio (( \zeta ))</td>
<td>( \zeta = \frac{1}{\pi} \ln \left( \frac{a}{b} \right) )</td>
<td>( \zeta = \frac{1}{2\pi} \ln \left( \frac{a}{c} \right) )</td>
</tr>
</tbody>
</table>

In order to calculate the Input Shaping coefficients enough to read the first two maximums of the Step Response Reaction ('a' and 'b' – see Fig.4) without calculation of
damping ratio. It’s possible to calculate all shaper coefficients for every shaper from table 1, when \( K = \frac{b}{a} \).

4. PROGRAMS AND SIMULATIONS

The MATLAB simulations of this paper were at 3 main concepts. First of all three main shapers were presented (Fig.5) and the simple ZV shaper was with the shortest command time but less robustness.

![Figure 5](image)

**Figure 5:** Commands and system reaction of three main zero vibration shapers

Second is comparing two Convolved and Direct Method get next plot (Fig.6):

![Figure 6](image)

**Figure 6:** Shaping for double pendulum systems – convolved and direct method

ZV-ZV by Convolved Method is faster then Direct Method.

Very important part of the Input Shaping method is to employ algorithm in practical situation. Many practical systems are required to fit in possible changes in process and use adaptation to various cases. Input command and parameters of system can interfere on Shaper. Adaptation algorithm should cope with different obstacles.

Below are 3 simple ways to use IS in adaption process. They are divided on:
- Idle step adaptation, with learning once and next system operation (Fig.7 and Fig.8)
- Step-by-Step adaptation, every step Shaper estimation (Fig.9)
- Internal adaptation, Shaper known immediately after beginning motion (Fig.10)
Every adaptation method was simulated with low robust shaper – ZV (Zero Vibration shaper). Any other shaper type will just improve the system performances.

**Figure 7:** Idle step adaptation – training

**Figure 8:** Idle step adaptation – shaped motion (no vibration)

**Figure 9:** Step-by-step adaptation – schematic diagram
The vibration system for experiment includes four main parts: Linear Pendulum, Motor, Drive and Graphical User Interface (GUI). The Linear Pendulum is dynamically similar to the crane. The system can be ready to exploitation at short time. It is pretty well for presentations. The general form of the experimental system is presented below (Fig.11).

The practical model was done by the recording of the pendulum vibration (See Fig.12). Vertical axis is Residual Vibrations in counts and horizontal – time in milliseconds.

Table 4 shows the needful information of the system. The coefficients are calculated by Two Points and Command method. The first and second pulse coefficients help to reduce the residual vibrations.
According to the Pendulum System data the experiment was performed. The results of the experiment are presented in the Table 5.

The motion is Point-to-Point. The steps were done for different distance with different speeds. Distance as shown in row ‘Position’ (in encoder counts) and velocities as said in ‘Velocity’ row (in Revolutions per minutes).

**Table 4: Pendulum System data**

<table>
<thead>
<tr>
<th>Data</th>
<th>Definition</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>First Extremum amplitude [Counts]</td>
<td>a</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>First Extremum time [Seconds]</td>
<td>t_a</td>
<td>0.906</td>
</tr>
<tr>
<td></td>
<td>Second Extremum amplitude [Counts]</td>
<td>b</td>
<td>-37</td>
</tr>
<tr>
<td></td>
<td>Second Extremum time [Seconds]</td>
<td>t_b</td>
<td>1.484</td>
</tr>
<tr>
<td></td>
<td>Third Extremum amplitude [Counts]</td>
<td>c</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Third Extremum time [Seconds]</td>
<td>t_c</td>
<td>2.069</td>
</tr>
<tr>
<td></td>
<td>Average Level [Counts]</td>
<td>N</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Half Period of Vibration [Seconds]</td>
<td>T/2</td>
<td>0.583</td>
</tr>
<tr>
<td></td>
<td>System Angular Frequency [Radians per Seconds]</td>
<td>ω</td>
<td>5.403</td>
</tr>
<tr>
<td></td>
<td>Damping Ratio [No Units]</td>
<td>ζ</td>
<td>0.012166</td>
</tr>
<tr>
<td></td>
<td>Percentage Overshoot [%]</td>
<td>K</td>
<td>96.25</td>
</tr>
<tr>
<td></td>
<td>First coefficient of IS filter [No Units]</td>
<td>A1</td>
<td>0.5095</td>
</tr>
<tr>
<td></td>
<td>Second coefficient of IS filter [No Units]</td>
<td>A2</td>
<td>0.4905</td>
</tr>
</tbody>
</table>

**Table 5: Input Shaping Experiment**

<table>
<thead>
<tr>
<th>Position [Counts]</th>
<th>Velocity [rpm]</th>
<th>5000</th>
<th>10000</th>
<th>20000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 200 500</td>
<td>100 200 500</td>
<td>100 200 500</td>
<td></td>
</tr>
<tr>
<td>Vibration [counts] - No Shaping</td>
<td>61 69 70</td>
<td>86 124 134</td>
<td>42 171 242</td>
<td></td>
</tr>
<tr>
<td>Vibration [counts] - With Shaping</td>
<td>2 2 4</td>
<td>3 3 4</td>
<td>8 8 20</td>
<td></td>
</tr>
</tbody>
</table>

Both rows in Table 5 show the results of the residual vibrations (in encoder counts) with IS algorithm and without it. For example the step 10000 [counts] with the velocity of 200 [rpm] will react with vibrations of 124 [counts]. If IS algorithm works (by dividing 10000 step for two smaller moves 10000*A1 and 10000*A2 with delay between for ‘T/2’), the motion reach the target with tiny vibration of 3 [counts], almost without vibrancy at all. Clearly seen the improvement with residual vibrations when IS algorithm is operated.

6. CONCLUSIONS

Several types of input shaping have been presented. The simple method of the shaper coefficients calculation can help in fast estimation of the system. Three adaptation processes give good results of the residual vibrations. “Idle step” and “Step-by-Step” adaptation calculation is in the position dimension and “Internal adaptation” use velocity data. The successful simulation shows that ZV shaper is good enough for practical experiments and possible to use ZV-ZV Convolved Method in Multimode systems. Pendulum system experiments get well results.
7. REFERENCES


