Accurate Calculation of Punching Control Perimeter in RC Flat Slabs and Flat Foundations

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Abstract
The punching problem of reinforced concrete flat slabs has two main aspects, related to load carrying capacity of these structures under shear forces: obtaining the shear force value, and finding the control area (failure zone dimensions). Generally, the punching shear force is considered according to experimental data and the control perimeter is assigned according to existing design codes. The available experimental and finite element analysis data differs by 40-50% from the values, given in modern design codes. The present study deals with exact evaluation of the control perimeter under shear punching. This study emphasizes the difference between punching shear due to concentrated load and support reaction force to distributed one. These both cases are investigated using mini-max principle: the internal shear forces are maximized and the control perimeter dimension leads to minimal external load. The obtained results can be used as a basis for refinement of existing punching shear models and further development of modern design provisions in this field.

Keywords: punching shear, flat slab, flat foundation, mini-max principle, reinforced concrete

1. Introduction
Punching of reinforced concrete (RC) slabs is investigated many years [1, 2]. A model for design of flat slab –joints to punching under symmetric loading was proposed by Kinnunen [1]. The size effect in punching shear strength was described by an approximate formula [3]. Punching shear models are based on empirical and simplified analytical techniques or on theory of plasticity [4].

A method for punching shear design of slab - column connections, subjected to seismic loading, was proposed [5]. The punching shear design considers the probable unbalanced bending moment in the connection, promoting possibility of flexural failure mode over a shear one. A modified yield line approach was presented. The predictions of probable unbalanced moment were compared with available results from the literature.

For analyzing punching shear behavior of RC structures, a two dimensional equivalent continuum model was extended to three dimensional finite element formulation [6]. It was concluded that a crack angle \( \Theta \) value of 45° is a good approximation for the average of all crack surface roughness. As concrete stress-strain curves shift from a uniaxial pattern, the concrete peak stresses in three directions were selected correspondingly. It was reported that the numerical results are in good agreement with available experimental data on load – deflection behavior of RC slabs [7]. In this test a hexagonal specimen, supported by 12 rods, was loaded at the center by a hydraulic jack. No shear reinforcement was provided in the slab.
A theoretical background to punching shear provisions of the *fib* Model Code for Concrete Structures [8] and an example of their application were presented [9]. The mechanical model that forms the basis for the punching design equations was explained. The relevance of the provisions was justified and their suitability for structural design was demonstrated.

To develop the design guidelines and accurate prediction of steel fibred reinforced concrete (SFRC) flat slabs punching resistance, a database from 154 punching tests was used [10]. The proposed method is based on the critical shear crack theory that is suitable to investigate the strength of slabs with shear reinforcement [11]. The method is capable to predict the slab load - rotation behavior, considering recommendations of CEB-FIP Model Code for modelling post-cracking behavior of SFRC [12].

Research was performed to study shear mechanisms that govern the behavior of RC structures subjected to localized impact loads [13]. Such phenomenon is due to combination of inertial and material strain-rate effects, leading to a stiffer slab behavior under higher loading rates. It can also lead to pure punching shear failure, instead of flexural one. The approach that was proposed considers the dynamic punching shear capacity, inertial and material strain-rate effects and demonstrated good correlation with experimental data.

Although there is a lot of experimental and numerical data on flat slab – column joints, similar investigations of flat foundation – column joints is rather limited. Behavior of RC column footing, laid on deformable subgrade and loaded by concentrated load until failure, was recently modeled and analyzed [14]. Field test data were used for model calibration. Comparison of the experimental and numerical results showed good agreement, but also revealed some questions regarding finite elements analysis.

As known, there are three types of internal forces in the punching failure zone (between the control perimeter and the column): shear forces, radial and tangential bending moments [1]. The influence of these forces on stress-strain state of the plate within the control perimeter was investigated [15]. It was assumed that the control perimeter passes through a section with zero radial moments. In this case, the shear forces contribution (weight coefficient) is about 85% of the total external load and the remained 15% is the weight coefficient of tangential moments. The control perimeter crack inclination angle relative to horizontal plane is assumed 35 … 45°. However, the critical control perimeter value was suggested to be half of the slab thickness. It should be mentioned that those assumptions are based on engineering experience and are not proved.

**2. Analysis of modern codes provisions on punching**

The punching problem includes various cases from the viewpoint of possible static schemes of flat slabs and foundations, resisting to punching (Figure 1), and available approaches, related to this problem (Table 1). Part of research results, related to this issue, were discussed in the previous chapter [1 – 15]. This chapter analyzes the punching problem from the viewpoint of modern design codes.

The current codes [8, 12, 16-18] are mainly focused on obtaining the value of punching shear force and control perimeter dimensions, as shown in Figure 2. According to modern design provisions, the punching zone control perimeter is symmetric in case of symmetric external forces. Shear forces and radial moments act along this perimeter. In these codes shear forces are calculated using empirical coefficients, which are different. It is because concrete shear strength in each code is determined, based on different
experimental investigations and assumptions. For example, in building rules (BR) [16], concrete shear strength is assumed equal to its tensile one. In our opinion, it is because there exact data on concrete shear strength is not available. The static scheme for punching calculation according to this code corresponds to Figure 1a.

Table 1. Available data on crack inclination angle and control perimeter

<table>
<thead>
<tr>
<th>No.</th>
<th>Code or reference</th>
<th>Static scheme (following Figure 1)</th>
<th>Application suitability</th>
<th>Limits of angle $\theta$, $^\circ$</th>
<th>Distance between control perimeter, $u$, and column</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BR [16]</td>
<td>1a</td>
<td>Slabs and foundations</td>
<td>26.6</td>
<td>0.5 $d$</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>EC 2 [17], CEB [12], IS 466 – 1 [18]</td>
<td>1c</td>
<td>Slabs</td>
<td>25 … 45</td>
<td>2.0 $d$</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>IS 466 – 1 [18]</td>
<td>1d</td>
<td>Foundations</td>
<td>-</td>
<td>1.0 $d$ $h = \text{const}$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>EC 2 [17]</td>
<td>1e</td>
<td>Foundations</td>
<td>$\geq \arctan 0.5$</td>
<td>- $h = \text{various}$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Model Code 2010 [8], Muttoni et al. [9]</td>
<td>1b</td>
<td>Slabs</td>
<td>-</td>
<td>0.5 $d$</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>Ahmad et al. [6]</td>
<td>1a</td>
<td>Slabs</td>
<td>45$^*$</td>
<td>1.0 $d^*$ FE mesh</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Yankelevsky et al. [4]</td>
<td>1a</td>
<td>Slabs</td>
<td>25 … 32</td>
<td>slightly less than 2.0 $d$</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>Reiss [15]</td>
<td>1c</td>
<td>Slabs</td>
<td>35 … 45</td>
<td>0.5 $d$</td>
<td>-</td>
</tr>
</tbody>
</table>

Notations: $u$ – control perimeter; $d$ – effective flat slab section height; $h$ – foundation plate thickness; $\theta$ – control perimeter crack inclination angle relative to horizontal plane.

* The values were calculated by the authors, based on a figure, presented in [6].

Another problem is selecting the control perimeter location, which has a wide spread of values (Table 1). Following the table, this spread is between 0.5 $d$ and 2.0 $d$, where $d$ is the effective flat slab section height. The control perimeter radial crack inclination angle relative to horizontal plane, $\theta$, varies from 19$^\circ$ to 45$^\circ$. The code recommends to use the value of 0.5 $d$ (corresponding to $\theta = 25.56^\circ$).
Following EC 2 [17], the control perimeter is 2.0 $d$. The code assumes static schemes, corresponding to Figure 1c and e for slabs and foundations, respectively. For slabs and foundations with variable thickness (raw 4 in Table 1) $\theta \geq \arctan 0.5$ ($\theta \geq 25.56^\circ$). For concentrated external force, $V_d$, the reduced load, applied to the foundation...
plate, \( V_{d, \text{red}} \), is decreased by the force, \( \Delta V_d \), caused by soil contact pressure within the control perimeter, i.e.

\[
V_{d, \text{red}} = V_d - \Delta V_d
\]

(1)

The code assumes that the distance \( 2.0 \, d \) that characterizes the control perimeter determines the minimal resistance of the plate to the external force.

![Figure 2. A general scheme for calculations, related to punching problem](image)

Following the above-mentioned codes, it can be concluded that the punching control perimeter is not calculated, but assumed within \((0.5 \ldots 2.0) \, d\). The expressions for concrete shear bearing capacity and the links along the control perimeter are empirical and include many coefficients, based on available experimental data. The existing codes do not define any difference between flat slab–column and column–flat foundation problems, as shown in Figure 1: in the first case there is a concentrated load, and in the second one – a uniformly distributed reaction.

### 3. Aims, scope and novelty

According to available publications and normative documents, dealing with punching problem, there are no strong analytical dependences for calculating the control perimeter location. It causes uncertainties regarding the shear links’ location. Usually shear links are placed according to engineering experience and the available experimental data [19].

The present study is focused at finding an accurate solution of the punching shear problem, related to the control perimeter value. Strong analytical dependencies are
proposed with this aim. Numerical examples demonstrate efficiency of the proposed approach and the analytical results are compared with available experimental and finite element analysis data.

In this study, two main calculation schemes, related to punching shear problem, are considered:
- external load that acts on a slab with concentrated reactions (see Figure 1a, b, c and f);
- external load that acts on a slab with uniformly distributed reaction (see Figure 1d and e);

The paper deals with analysis of the above-mentioned calculation schemes. It enables to solve the punching problem separately for the cases of flat slab and flat foundations. For solving this problem, mini-max principle was applied [20, 21] considering ultimate limit state analysis [22].

At the same time, taking into account that punching failure is brittle, it is proposed to use steel fibers within the punching control perimeter. Efficiency of using steel fibers in preventing brittle failure of concrete was demonstrated previously [23, 24].

4. Main assumptions and solution
To allow proper analysis of available data for punching shear calculations, the following main assumptions can be used:
- the vertical concentrated load that yields punching, acts on the slab without eccentricity, i.e. the control perimeter is symmetric relative to the column;
- there is no shear reinforcement, i.e. concrete takes the whole shear force;
- the influence of tangential bending moments is neglected;
- the slab failure due to radial bending moments is excluded;
- the value of $V_{Rdc}$ is known and taken according to [16], suggesting that the concrete shear resistance is equal to its tensile strength.

5. Conclusions
Punching of reinforced concrete flat slabs and flat foundations has two main aspects: obtaining the shear force value, and finding the control perimeter that defines the failure zone dimension. According to most available methods, the control perimeter is assigned and not calculated, but the problem is that this perimeter is different for flat slabs and flat foundations. Moreover, available data, used in modern design codes for assigning this perimeter, has a wide scatter (up to 40-50%).

In the frame of the present study, a method for evaluation of the control perimeter under shear punching in flat slabs and flat foundations is proposed. The difference between punching shear due to concentrated load and support reaction force due to distributed one is shown. Both cases are investigated using mini-max principle, the concept of which is described in the paper. For analyzing the punching shear capacity, the static method of structural ultimate equilibrium is used. The kinematic parameter is taken, based on the most critical section from the punching shear viewpoint.

The proposed method allows calculation of the control perimeter dimension and avoids the need for assuming it, like in existing design codes. As punching failure is brittle, it is proposed to add steel fibre within the control perimeter. It is also shown that
the proposed methodology can be applied for calculating flat foundations with variable thickness.

Efficiency of the proposed method is demonstrated in numerical examples. The obtained results are compared with available experimental and numerical data. It is shown that the proposed method enables to predict the control perimeter dimension and the ultimate load on the flat slab and flat foundation with high accuracy. It allows finding a more exact location of shear links in flat slabs, if it is necessary according to calculation.

The proposed method can be used as a basis for improving the existing punching shear models and further development of modern design provisions for punching.

References
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