ARIEL UNIVERSITY PRESS

Functional Differential Equations
Volume 31, No. 3-4, 2024, pp. 135-143
DOI: https://doi.org/10.26351/FDE/31/3-4/1
ISSN: 0793-1786 (print), 2617-8605 (online)

PROFESSOR LINA RAKHMATULLINA (1932 – 2024)

E. BRAVYI , A. DOMOSHNITSKY , I. KIGURADZE , V. MAKSIMOV AND V. MALYGINA

On March 19, 2024, Lina Fazylovna Rakhmatullina, a known mathematician, a founder of the modern theory of linear functional differential equations, Doctor of Physical and Mathematical Sciences, Professor, Honored Worker of the Higher School of the Russian Federation, passed away at the age of 92.

Lina Fazylovna was born on January 3, 1932 in the village of Baltasi in Tatarstan, where her parents were employees of the district administration. After graduating from high school with a medal in 1949, she entered the mathematical department of the Physics and Mathematics Faculty of the Kazan State University, from which she successfully graduated in 1954. Her thesis was published in the academic notes of Kazan University. In the 1954-1955 academic year, she worked as a mathematics teacher at a school for working youth in the Vologda region. From September 1955 to admission to a one–year postgraduate course in 1960, she was an assistant at the Udmurt State Pedagogical Inst UdSPI itute (). Lina Fazylovna defended her PhD thesis on the stability of differential equations in 1963 at Kazan University. After finishing postgraduate studies, she continued her work until September 1965, first as a teacher, then as an associate professor at UdSPI. She has been working since 1965 as a senior lecturer, and then as an associate professor of the Department of Higher Mathematics at the Izhevsk Mechanical Institute, where, with her active participation, the specialty "mathematical engineer" was opened.

In 1966, together with her husband N.V. Azbelev, she moved to Tambov, taking up the position of associate professor at the Tambov Institute of Chemical Engineering. Here she organized groups of students with advanced mathematical training. In Tambov, L.F. Rakhmatullina actively studied linear equations with a deviating argument and introduced a modern definition of the concept of solution for them. This definition formed the basis of a new concept of equations with a deviating argument. In Tambov, L.F. Rakhmatullina and N.V. Azbelev organized a seminar on functional differential equations. The mentioned concept allowed the seminar to develop a general theory of linear functional differential equations, some classes of which were previously studied out of connection with each other. The foundation of the new theory, which was vigorously developing in the following decades, were the results of research by Lina Fazylovna and her graduate students M.P. Berdnikova and V.P. Maksimov.

The talent of a research scientist and the bright pedagogical talent of Lina Fazylovna were recognized and appreciated by colleagues at the department: Associate Professor Rakhmatullina has consistently proved to be the main authority on difficult issues of mathematics and a patient consultant in the preparation of reports and manuscripts of scientific articles. At the invitation of the rector of the Perm Polytechnic Institute, Professor M.N. Dedyukin, L.F. Rakhmatullina and N.V. Azbelev, together with a large group of participants of the Tambov seminar, moved to Perm in late 1975, forming a new department of Mathematical Analysis at the Institute.

Lina Fazylovna worked as an associate professor and then professor of this department until 1994, when the department was liquidated and the Research Center "Functional Differential Equations" (RC FDE) was founded, where Lina Fazylovna worked until 2012. In 2006, after the death of Nikolai Viktorovich, Lina Fazylovna took over the leadership of the center.

Lina Fazylovna defended her doctoral dissertation in Kiev at the Institute of Mathematics of the Academy of Sciences of Ukraine in 1982. Dissertation on linear functional differential equations aroused great interest and heated discussions related to the rejection of the proposed new concept of equations with a deviant argument by some Moscow mathematicians. After defending her doctoral dissertation, Lina Fazylovna continued her research and teaching work with renewed vigor. Under her supervision and due to her consultation, the doctoral dissertations of V.P. Maksimov and L.M. Berezansky and 12 PhD dissertations were completed.

In 1991, the publishing house Nauka in Moscow published a monograph by "Introduction to the Theory of Functional Differential Equations" by L.F. Rakhmatullina in collaboration with N.V. Azbelev and V.P. Maksimov. This monograph, republished in 1995 in English in the USA, systematizes the research results of the Tambov and Perm seminars. In 1996, the Publishing House GCI in Tbilisi published a monograph by N.V. Azbelev and L.F. Rakhmatullina "Theory of Linear Abstract Functional Differential Equations and Applications". The object of the theory of abstract functional differential equation (AFDE) described in it are equations with a phase space D isomorphic to the direct product of a Banach space B and a finite-dimensional space R^m (D = B x R^m). Within the framework of the general theory of AFDE, results on boundary value problems, control problems, variational problems and problems of solution stability are obtained. The central idea of the applications of the AFDE theory is the rational choice of the space D for each specific class of models and each of the mentioned tasks. Such a choice, in the presence of a general theory, allows ones to apply standard schemes and theorems of analysis to problems whose study previously required an individual approach and special constructions. Considerable freedom of choice of the space D has made it possible to cover the following classes of functional differential equations with ordinary derivatives of the whole order with meaningful results: systems with aftereffect, singular systems, systems with impulse action, hybrid systems.

In 2002, the monograph "Elements of the Contemporary Theory of Functional Differential Equations. Methods and Applications" was published in the Moscow publishing house "Institute of Computer-Assisted Research" in collaboration with N.V. Azbelev and V.P. Maksimov, in which the main

statements of the general theory of functional differential equations are presented systematically from a uniform point of view based on the fundamental statements of the Theory of AFDE. The book aroused wide interest among specialists, its English-language version "Introduction to the Theory of Functional Differential Equations: Methods and Applications" was published in 2007 by Hindawi Publishing Corporation, New York in the series Contemporary Mathematics and Its Applications. Lina Fazylovna's contribution to the creation of the AFDE Theory cannot be overestimated.

Lina Fazylovna's scientific work is characterized by the originality of the formulation of tasks and the impeccable rigor of proofs. Her teaching activity was characterized by high demands, a friendly attitude towards students and thorough preparation for all types of classes. Lina Fazylovna's authority among colleagues was very high.

Lina Fazylovna was a passionate traveler, she and Nikolai Viktorovich spent every vacation traveling around Russia, Kazakhstan, Kyrgyzstan, the Caucasus, first on motorcycles, later on a Moskvich or off-road vehicle LUAZ.

Being a great lover and connoisseur of opera and ballet, Lina Fazylovna successfully promoted among her colleagues and students the famous Perm Opera and Ballet Theater, with many of whose artists she was connected by sincere friendship.

Lina Fazylovna's whole life is a vivid example of selfless service to science and mathematical education. For all students and junior colleagues, she will remain in the memory of a Teacher who largely determined the attitude to science and life.

The name of Lina Fazylovna Rakhmatullina will always remain in the history of functional differential equations theory development, and her memory will live in our hearts.

Selected Bibliography of L.F. Rakhmatullina

Monographs

1. Introduction to the theory of functional differential equations (with N.V. Azbelev and V.P. Maksimov). (Russian) Nauka, Moscow, 1991.

2. Introduction to the theory of linear functional differential equations (with N.V. Azbelev and V.P. Maksimov). Atlanta: World Federation Publishers Company, 1995.

3. Theory of linear abstract functional differential equations and applications (with N.V. Azbelev). Mem. Differential Equations Math. Phys. Tbilisi: Publishing House GCI, 1996.

4. Methods of the contemporary theory of linear functional differential equations (with N.V. Azbelev and V.P. Maksimov). (Russian) Regular

and Chaotic Dynamics, Moscow, Izhevsk, 2000.

5. Methods of the contemporary theory of functional differential equations. Methods and applications (with N.V. Azbelev and V.P. Maksimov). (Russian) Institute for Computer-Assisted Research, Moscow, 2002.

6. Introduction to the Theory of Functional Differential Equations: Methods and Applications (with N.V. Azbelev and V.P. Maksimov). Hindawi Publishing Corporation, New York, 2007.

Journal Articles

1. On a class of nonlinear singular integral equations (Russian). Uch. Zapiski of the Kazan University, 1955, v. 115. No 7, 25-29.

2. On an extension of the solution of the Chaplygin Theorem outside the feasibility boundary of the differential inequality theorem (with N.V. Azbelev and Z.B. Tsalyuk). (Russian) Nauchn. Dokl. Vyssh. Shkoly. Phys. And Math., 1958, No 2, 3-5.

3. Note on the positivity of inverse operators (with N.V. Azbelev and Z.B. Tsalyuk). (Russian) Uch. Zap. of the Udm. State Ped. Inst., 1958, No 12, 47-49.

4. To the question of application of the straight lines method. (Russian) Uch. Zap. of the Udm. State Ped. Inst., 1958, No 12, 65-68.

5. On an application of solvability conditions of the Chaplygin theorems to the questions of boundedness and stability of solutions to differential equations. (Russian) Izv. Vyssh. Uchebn. Zavrd. Mat., 1959, No 2, 198-201.

6. On the problem of stability of the solution to the nonlinear heat-conduction equation. (Russian) Prikl. Math. Mekh., v. 25, issue 3, 1961, 591-592; translation in PMM, J. Appl. Nath. Mech. 25(1961), 880-883.

7. To the question of linear systems stability. (Russian) Volzhsk. Mat. Sbornik, 1966, No 3, 310-313.

8. Theorems of solutions existence and convergence of successive approximations for nonlinear integral equations with deviating argument (with N.V. Azbelev and M.P. Berdnikova). (Russian) Trudy Tambov Inst. Chem. Engineering, 1969, No 3, 65-68.

9. Integral equations with deviating argument (with N.V. Azbelev and M.P. Berdnikova). (Russian) Dokl. Acad. Nauk. SSSR, 1970, v. 192, No3, 472-482; translation in Sov. Mat., Dokl. 11(1970), 643-647.

10. On the question of reducing a linear integral equation with deviating argument to the Fredholm equation (with M.P. Berdnikova). (Russian) Trudy Tambov Inst. Chem. Engineering, 1970, No 4, 64-65.

11. Existence, uniqueness and convergence of successive approximations for nonlinear integral equations with deviating argument (with N.V. Azbelev and F.I. Chigirev). (Russian) Differentsial'nye Uravneniya, 1970, v. 6, No 2,

223-229.; translation in Different. Equat., 6(1972), 173-178.

12. On linear equations with deviating argument (with N.V. Azbelev). (Russian) Differentsial'nye Uravneniya, 1970, v. 6, No 4, 616-628; translation in Different. Equat., 6(1973), 473-482.

13. W-method in the theory of differential equations with deviating argument (with N.V. Azbelev and A.G. Terent'ev). (Russian) Trudy Tambov Inst. Chem. Engineering, 1970, No 4, 60-63.

14. To the theory of linear equations with functional argument. (Russian) Differentsial'nye Uravneniya, 1972, v. 6, No 4, 616-628;

15. The Cauchy problem for differential equations with retarded argument (with N.V. Azbelev). (Russian) Differentsial'nye Uravneniya, 1972, v. 8, No 9, 1542-1552; translation in Different. Equat., 8(1972), 1190-1198.

16. The representation of the solution to a linear differential equations with retarded argument (with N.V. Azbelev). (Russian) Trudy Moscow Inst. Chem. Engineering, 1972, issue 39, 7-10.

17. On the dimension of the solutions space of linear differential equations. (Russian) Trudy Moscow Inst. Chem. Engineering, 1972, issue 39, 10-11.

18. On the question of regularization to linear boundary value problems (with A.G. Terent'ev). (Russian) Differentsial'nye Uravneniya, 1973, v. 9, No 5, 868-873; translation in Different. Equat., 9(1975), 660-664.

19. On a representation of solutions to linear functional diferential equations (with V.P. Maksimov). (Russian) Differentsial'nye Uravneniya, 1973, v. 9, No 6, 1026-1036; translation in Different. Equat., 9(1975), 781-790.

20. Linear functional differential equation solved with respect to the derivative (with V.P. Maksimov). (Russian) Differentsial'nye Uravneniya, 1973, v. 9, No12, 2231-2240.

21. On the question of continuous dependence on parameters for the solution of a differential equation with delay (with I.V. Shragin). (Russian) Trudy Moscow Inst. Chem. Engineering, 1973, issue 48, 65-66.

22. Conditions of the continuous dependence on parameters for the solution of the linear functional differential equation (with N.V. Azbelev). (Russian) Trudy Moscow Inst. Chem. Engineering, 1974, issue 53, 70—72.

23. A necessary and sufficient condition for the existence of The Green matrix (with A.G. Terent'ev). (Russian) Trudy Moscow Inst. Chem. Engineering, 1974, issue 64, 21-23.

24. On canonical forms of linear functional differential equations. (Russian) Differentsial'nye Uravneniya, 1975, v. 11, No 12, 2143-2153; translation in Different. Equat., 11(1976), 1589-1597.

25. On the linear functional differential equation of the evolutionary

type (with N.V. Azbelev and L.M. Berezankii). (Russian) Differentsial'nye Uravneniya, 1977, v. 13, No 12, 1915-1925; translation in Different. Equat., 13(1977), 1331-1339.

26. Linear functional differential equations (with N.V. Azbelev). (Russian) In: Differential equations with deviating argument, Naukova Dumka, Kiev, 1977, 11-19.

27. Interpolation method for approximate solution to linear boundary value problems (with N.V. Azbelev). (Russian) Boundary value problems, Perm State Univ., 1977, No 203, 53-56.

28. The adjoint equation for the general boundary value problem (with V.P. Maksimov). (Russian) Differentsial'nye Uravneniya, 1977, v. 13, No 11, 1966-1973; translation in Different. Equat., 13(1977), 1368-1373.

29. The adjoint equation for the functional differential equations of the n-th order. (Russian) Functional Differential and Boundary Value Problems of Math. Phys., Perm State Univ., 1978, 108-113.

30. Functional differential equations (with N.V. Azbelev). (Russian) Differentsial'nye Uravneniya, 1978, v. 14, No 5, 771-797; translation in Different. Equat., 14(1978), 547-565.

31. An Example of the interpolation to the Cauchy operator (with N.V. Azbelev and A.A. Efremov). (Russian) Functional Differential and Boundary Value Problems of Math. Phys., Perm State Univ., 1978, 43-47.

32. The Green operator and regularization of linear boundary value problem. (Russian) Differentsial'nye Uravneniya, 1979, v. 15, No 3, 425-435; translation in Different. Equat., 15(1979), 297-234.

33. Approximate construction of the Cauchy function (with N.V. Azbelev and A.A. Efremov). (Russian) Boundary Value Problem. Perm Polytech. Inst., Perm, 1979, 64-68.

34. Approximate construction of the general solution to the linear functional differential equation (with N.V. Azbelev and S.A. Yuganova). (Russian) Boundary Value Problem. Perm Polytech. Inst., Perm, 1980, 68-71.
.

35. Functional differential equations (with N.V. Azbelev and V.P. Maksimov). (Russian) Uspekhi Mat. Nauk, 1981,v. 36, No 4, 205.

36. On the Green of the general linear boundary value problem (with N.V. Azbelev and V.P. Maksimov). (Russian) Uspekhi Mat. Nauk, 1982, v. 37, No 4, 119-120.

37. On the reducibility of functional differential equations (with N.V. Azbelev and V.P. Maksimov). (Russian) Uspekhi Mat. Nauk, 1983, v. 38, No 5, 128-129.

38. Functional differential equations (with N.V. Azbelev and V.P. Mak-

simov). (Russian) The Ninth International conf. on nonlinear oscillations. Naukova Dumka, Kiev, 1984, V. 2, 13-16.

39. On definition of the notion of the solution to the equation with deviating argument. (Russian) Functional Differential Equations, Perm. Polytech. Inst., Perm, 1985, 13-19.

40. On the question of functional differential inequalities and monotone operators (with N.V. Azbelev). (Russian) Functional Differential Equations, Perm. Polytech. Inst., Perm, 1986, 3-9.

41. On regularization of linear boundary value problems. (Russian) Izv. Vyssh. Uchebn. Zaved. Mat., 1987, No 7, 37-43; translation in Sov. Mat., 31(1987), 46-55.

42. Abstract functional differential equation (with N.V. Azbelev). Functional Differential Equations, Perm. Polytech. Inst., Perm, 1987, 3-11.

43. Representation of solutions to linear functional differential equations which are not everywhere solvable. (Russian) Functional Differential Equations, Perm. Polytech. Inst., Perm, 1988, 10-13.

44. Theory of the abstract linear functional differential equation. (Russian) In: Questions of the qualitative theory of differential equations, Nauka, Novosibirsk, 1988,114-117.

45. The qeneralized Green operator of the overdetermined boundary value problem for the linear functional differential equation. (Russian) Izv. Vyssh. Uchebn. Zaved. Mat., 1993, No 5, 95-100; translation in Russian. Mat. (Iz. VUZ), 37(1993), 85-89.

46. On extension of the Vallee Poussin theorem to equations with aftereffect (with N.V. Azbelev). In: Boundary Value Problems for Funct. Different. Equat. World Scientific Publishing Co. Pte. Ltd, Singapur, 1995, 23-36.

47. The Vallee-Poussin theorem on differential inequality for equations with aftereffect (with N.V. Azbelev). (Russian) Trudy Mat. Inst. Steklov, 1995, v.211, 32-39; translation in Proc. Steklov Inst. Math. 211(1995), 28-34.

48. On an estimate of the spectral radius of the linear operator in the space of continuous functions (with N.V. Azbelev). (Russian) Izv. Vyssh. Uchebn. Zaved. Mat., 1996, No 11, 23-28; translation in Russian. Mat. (Iz. VUZ), 40(1996), 21-26.

49. Continuous dependence on parameters of the solution to the linear boundary value problem (with A.V. Anokhin). I (Russian) Izv. Vyssh. Uchebn. Zaved. Mat., 1996, No 11, 29-38; translation in Russian. Mat. (Iz. VUZ), 40(1996), 27-36.

50. The upper estimate of the spectral radius of the isotonic operator in the space of continuous functions. Memoirs on Different. Equat. Math.

Physics, 1997, v. 11, 163-167.

51. Stability of solutions of the equations with aftereffect (with N.V. Azbelev). Funct. Different. Equat., 1998, v. 5, No 1-2, 39–55.

52. The upper estimate of the spectral radius of the isotonic operator. (Russian) Izv. Vyssh. Uchebn. Zaved. Mat.,2000, No 1, 56-65; translation in Russian. Mat. (Iz. VUZ), 44(2000), No 1, 54-63..

53. Continuous dependence on parameters of the solution to the linear boundary value problem. (Russian) Izv. Vyssh. Uchebn. Zaved. Mat., 2000, No 4, 41-49; translation in Russian. Mat. (Iz. VUZ), 44(2000), No4, 39-47.

54. On the problem of convergence of solutions of linear boundary value problem. Funct. Different. Equat,. 2000, v. 7, No 3-4, 325—334.