Task 1

Four friends shared oranges from a box. The first took 1/4 of all oranges and 3 additional oranges from a box, the second took 1/3 of the remaining oranges and 2 more oranges, the third took half of the remaining oranges and another orange. As a result, the last friend got only 2 oranges. How many oranges were in the box?

Task 2

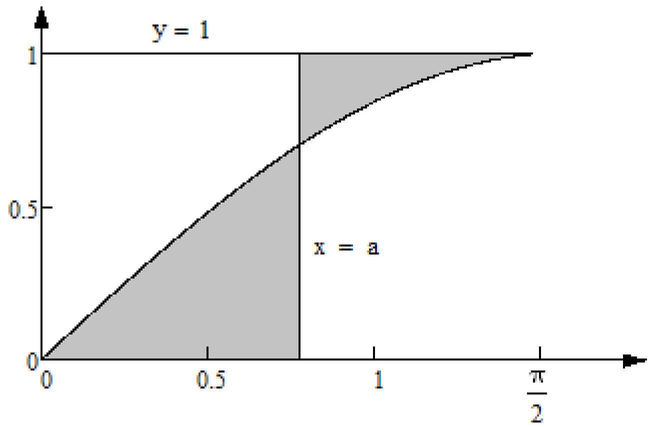
Find the limit $\lim_{a\to 0}\frac{\left[\frac{f'(a)\cdot f'(-a)}{\left(\frac{f(a)-f(-a)}{2a}\right)^2}-1\right]}{a^2}$, where $f(x)$ is an infinitely differentiable function, satisfying the following conditions: $f(a)\neq f(-a)$, $f'(0)=1$, $f''(0)=2$, $f'''(0)=3$, $f^{IV}(0)=4$, $f^{V}(0)=5$.

Task 3

It is known that the following six numbers $x, y, 189, 264, 287, 320$ (not necessarily arranged in ascending order) the set of all pairwise sums of four numbers a, b, c, d. Find the maximum possible value of $x+y$.

Task 4

For some value $a\in[0;\pi/2]$ the sum of areas of the two curve triangles, bounded by the graph of the function $y=\sin x$ and the lines $x=a, y=0, y=1$ (see the figure) is the minimum possible. Find the value of π/a.



Task 5

There are 2020 doctors in country N, each has his own illness. To choose an effective treatment, a doctor must be examined by all the other doctors. There is a need to use a pair of rubber gloves for each examination. If a doctor used gloves for examination, then the inside of the gloves becomes infected with the germs of his disease, and the outside — by the germs of the patient's disease. To save money on gloves, one can wear a glove on another and also turn a glove inside out.

What is the minimum number of pairs of gloves required for this?

Task 6

A dog is chained to a tree located at the origin. Let **i** be the unit vector of the *x* axis, and let **j** be the unit vector of the *y* axis. Initially, the dog is on the positive part of the *x* axis 100 yards from the tree (coordinates (100; 0)). From this point, the dog goes straight in the direction of the vector j. After passing 100π yards, it turns 90^o to the left and passes $\frac{100\pi^2}{2!}$ yards. Then it turns 90^o to the left and passes $100\cdot\frac{\pi^3}{3!}$, and so on, each time turning 90^o to the left and passing the *n*-th section of the $100\cdot\frac{\pi^n}{n!}$ yards long ($n=1,2,3,\ldots$). What are the coordinates of the limit point as $n\to\infty$?

Task 7

What is the number of 2×2 matrices $A\in M_2(R)$ with real entries, such that $A^2=\begin{pmatrix}1 & 1\\ 0 & 4\end{pmatrix}$?

Task 8

A monic polynomial $P_n(x)$ of degree *n* with integer coefficients can be factorized as $P_n(x)=(x-x_1)(x-x_2)\ldots(x-x_n)$, where $0<x_k<3$ for all $k=1,2,\ldots,n$.
Find all the possible values of x_k for $k=1,2,\ldots,n$. For the computer please enter only the number of such possible values.

Task 9

In the space there are several planets having the shape of a ball, each of which has radius $\frac{10}{\sqrt{\pi}}$, expressed in astronomical units. On each planet, we mark the set of points from which no other planet is visible. Find the sum of the areas of the sets of marked points on all the planets, expressed in square astronomical units.

Task 10

How many positive integers *k* are there such that for all positive integers *n* the number $k\cdot 2^n+1$ is composite.
(Enter the letter A, B, C or D into the system)
Answer Options:
A) 0,
B) 1
C) 10
D) infinity